ESSAYS ON THE MONETARY POLICY TRANSMISSION MECHANISM

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ESSAYS ON THE MONETARY POLICY TRANSMISSION MECHANISM

My thesis studies the effect of financial frictions on the monetary policy transmission mechanism in four chapters.

The first chapter, *Credit Constraints in a New Keynesian Framework: A Simple Theoretical Analysis*, studies how financial frictions alter equilibrium outcomes. I show that collateral borrowing constraints have general equilibrium effects that operate via four channels; borrowers' collateral value, savers' portfolio reallocation, wage adjustments, and wealth effects due to changes in firms' profits.

In the second chapter, *Monetary Policy and Credit Constraints,* I present an extension of the model in the first chapter and calibrate the baseline version of this model to the U.S. economy. I show that a monetary expansion leads to a smaller increase in aggregate output in the New Keynesian model where the collateral borrowing constraints are included relative to the standard model without constraints. As the fraction of credit constrained agents becomes larger, the increase in aggregate output becomes smaller.

In the third chapter, *An Empirical Assessment of the Transmission of Monetary Policy,* I conduct a study of the transmission of monetary policy via three credit costs components; risk-free rate (conventional channel), spread over the risk-free rate (credit spread channel), and non-spread factors such as credit limits (non-spread credit channel). I find that financial frictions are relevant for monetary policy transmission; the credit channel accounts for about 20% - 30% of the change in consumption expenditures. Additionally, non-spread factors play a non-trivial role.

In the final chapter, *Monetary Policy Revisited: Heterogeneous Bank Pass-Through of Credit Expansions*, I relax the assumption that the central bank directly impacts

borrowing/lending rates by explicitly modeling the pass-through from banks to households. In my model, banks offer differentiated credit contracts to households as a consequence of financial frictions. Following monetary expansions, the relaxation of credit conditions is about three times larger for households at the top of the wealth/income distribution relative to those at the bottom. Incorporating this heterogeneous pass-through mechanism reduces the response of aggregate consumption to a monetary expansion by about five times.

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CONTENTS

CHAPTER 1

Credit Constraints in a New Keynesian Framework: A Simple Theoretical Model

Abstract

Using a simple two-period, two-agent version of the baseline two-sector New Keynesian model, I provide a theoretical illustration on how financial frictions alter equilibrium outcomes. I introduce the financial friction as a collateral constraint faced only by one type of agents; the borrowers. The other type of agents, the savers, don't face a collateral constraint and they have some ownership of the firms in the economy. I show that under this set up, borrowing constraints have general equilibrium effects that operate via four channels; borrowers' collateral value, savers' portfolio reallocation, wage adjustments, and wealth effects due to changes in firms' profits.

JEL Codes: E21, E44

1.1 Introduction

The basic New Keynesian model has become the workhorse for analyzing and understanding the effects of different monetary policy interventions in the macroeconomy. Various authors have extensively studied the macroeconomic implications of the monetary authority in simple versions of the New Keynesian model, which includes authors such as Christiano and Evans (2005), Clarida, Galí and Gertler (2000), Galí (2008), Woodford (2001), and Woodford (2003), just to mention a few.

However, after the 2008 financial crisis, several authors have brought into attention some shortcomings of the original simple New Keynesian framework. This literature has come to emphasize the increasing importance of the financial sector in the macroeconomy; the interactions between the financial sector and monetary policy can no longer be ignored.

Motivated by these observations, I provide a theoretical illustration of the effect of financial frictions on equilibrium outcomes within a simple two-period, two-sector, standard New Keynesian model. The model features two types of agents who differ on their degree of patience, durable and non-durable consumption goods, nominal pricing frictions à la Calvo in the production sector of each good, a nominal bond, and a monetary authority which is introduced via a Taylor rule. The financial friction is introduced as a collateral constraint that limits borrowing via the nominal bond; the relatively more impatient agents must use the durable good as collateral to back their borrowing.

I show that borrowing constraints have general equilibrium effects that operate via four channels; borrowers' collateral value, savers' portfolio reallocation, wage adjustments, and wealth effects due to change in firms' profits. When the constraint is binding, the no-constraint allocation is no longer feasible for the less patient agents (borrowers). The adjustment in their allocation affects firms' profits. To the extent that the more patient agents (savers) have some ownership of the firms, the change in profits generates a wealth effect; they too must adjust their allocation. Given the adjustment in both agents' allocations, the wage rate, relative durable price, and nominal interest rate adjust as well to ensure markets clear.

For the borrowers, the general equilibrium price changes are relatively unimportant; they adjust their allocation mostly to ensure it is feasible given the constraint. However, for the savers, the general equilibrium price changes lead to a significant adjustment in their allocation. For instance, the adjustment in firms' profits, the durable's relative price, and the nominal interest rate are responsible for the savers' increase in the initial period's durable consumption. Furthermore, this increase in savers' durable consumption is the main driver of the increase in the initial period's aggregate durable output.

Thus, the main takeaway of the chapter is really simple; the credit constraint (and financial frictions for that matter) have important indirect general equilibrium effects. Furthermore, this indirect general equilibrium effects are not only important to the extent that they affect individual agents in the economy; they can also have an impact on aggregate outcomes.

The rest of the chapter is organized as follows. Section 1.2 specifies in detail each of the components of the model. In Section 1.3 the model's equilibrium is defined and a characterization of the equilibrium solution is presented. Section 1.4 presents the main results of the paper. It first provides a theoretical characterization of the partial equilibrium effects of the credit constraint and then provides a numerical characterization of the general equilibrium effects. Finally, the concluding remarks are presented in Section 1.5.

1.2 Model

The basic framework consists of a two-period, two-sector, New Keynesian model with price adjustments as in Calvo (1983). The economy is populated by two types of agents who differ along three key dimensions: their degree of patience, the ownership of firms, and their ability to access the financial market. Agents work, consume, and accumulate wealth using two instruments; a durable good and a nominal risk free bond. The agents with the smallest time preference discount parameter (*borrowers*), face a collateral credit constraint on their nominal debt holdings.[1] The agents with the largest time preference discount parameter (*savers*) can freely access the financial market.[2] The savers are the owners of the firms that produce the intermediate varieties. The intermediate firms use labor as the only input in production. There are two final consumption goods, durables and non-durables, which are produced using the intermediate varieties. Monetary policy is conducted through a nominal interest rate rule of the type first introduced by Taylor (1993). Finally, I assume

[1] This constraint is of the type proposed by Kiyotaki and Moore (1997).

[2] Note that despite the use of the terms borrowers and savers, both types of agents can choose to borrow or save using a nominal bond. I use this terminology given that in the model's solution the more patient agents will lend resources to the impatient agents.

there are no aggregate stochastic shocks.

1.2.1 Producers of Final Goods

Denote the consumption goods by $j \in \{c, d\}$; where c stands for non-durable good and d for durable. Firms in sector j operate under a perfectly competitive environment. For each period $t \in \{0, 1\}$, a firm in sector j produces the final good using as inputs a continuum of differentiated goods; which it buys from the intermediate firms. Let these intermediate goods be indexed by $i \in [0, 1]$; $y_{j,t}(i)$ is the i^{th} intermediate variety to be used in the production of the consumption good $Y_{j,t}$ at period t. The price a producer must pay for the intermediate variety $y_{j,t}(i)$ is just $P_{j,t}(i)$.

All firms within sector j have access to the same technology, which is given by the CES production function

$$Y_{j,t} = \left(\int_0^1 y_{j,t}(i)^{\frac{\epsilon_j - 1}{\epsilon_j}} di \right)^{\frac{\epsilon_j}{\epsilon_j - 1}}, \tag{1.2.1}$$

where $\epsilon_j > 1$ is the sector specific elasticity of substitution between differentiated varieties. Firms sell the final good $Y_{j,t}$ to the borrower and the saver at price $P_{j,t}$.[3] Therefore, for a firm producing the final good for sector j, the profits at period t are given by:

$$\Pi^F_{j,t} = P_{j,t} Y_{j,t} - \int_0^1 P_{j,t}(i) y_{j,t}(i) di \tag{1.2.2}$$

For each $t \in \{0, 1\}$, given the consumption good prices $\{P_{j,t}\}$ and the intermediate varieties prices $\{P_{j,t}(i)\}_{i \in [0,1]}$, a firm producing the final good $Y_{j,t}$ must solve the problem of choosing the intermediate varieties $\left\{ y_{j,t}(i) \geq 0 \right\}_{i \in [0,1]}$ to maximize (1.2.2) subject to (1.2.1).

1.2.2 Producers of Intermediate Goods

There is a continuum of mass one of intermediate firms. Each intermediate firm manufactures a different variety to be used in the production of the final consumption good of sector j. Let these firms be indexed by $i \in [0, 1]$; then $y_{j,t}(i)$ refers to the intermediate variety manufactured by firm i used in the production of consumption good j at period t.

[3] Note that from the firms' perspective, this price is exogenous. The zero-profit condition that arises due the the perfectly competitive environment is what determines the price index $P_{j,t}$.

The only input in the production of varieties is labor. It is assumed that all intermediate firms within sector j have access to the same production technology. Furthermore, to keep the analysis tractable, such production technology is assumed to be linear in the labor input:

$$y_{j,t}(i) = A_j n_{j,t}(i), \tag{1.2.3}$$

where A_j is a constant that measures the sectorial labor productivity.[4] Since labor is perfectly mobile across sectors, and given that the labor market is perfectly competitive, all intermediate firms must pay the same nominal wage rate W_t. Note also that, from the perspective of the individual firm, this wage rate is exogenously given.

The intermediate firms operate under monopolistic competition; each firm chooses to charge a price $P_{j,t}(i)$ to the producers of the consumption good j given their demand for the variety $y_{j,t}(i)$. However, it is assumed that there is a random variable, $\mathbb{S}_j \sim$ Bernoulli (ϕ_j), which governs whether firm i in sector j is able to charge its price optimally or if it must charge the same price it charged the previous period.[5] This random variable is i.i.d. across both, intermediate firms (i) and time periods (t). Simply put, for any time period t, an intermediate firm within sector j may not reset its price with probability ϕ_j.

Finally, solely for tractability purposes, it is assumed that the intermediate firms are entirely owned by the savers and shares of these firms can not be traded. Given this assumption, the intermediate firms discount the future nominal profits at $t = 1$ using the savers' nominal stochastic discount factor:

$$\begin{aligned}\Lambda_1^s &= \frac{\lambda_1^s}{\lambda_0^s} \\ &= \beta_s \frac{c_0^s}{c_1^s} \frac{P_{c,0}}{P_{c,1}},\end{aligned} \tag{1.2.4}$$

where λ_t^s is the shadow price of a unit of the nominal asset for the savers at period $t \in \{0, 1\}$.[6]

Given this specification, for an intermediate firm i in sector j that is able to reset its price at the

[4] In the current analysis, this constant is set to one in both sectors: $A_d = A_c = 1$.
[5] As in Calvo (1983).
[6] This shadow price is obtained by taking the FOC's of the saver's problem.

initial period, the present discounted value of the firm's profits are given by:

$$\Pi_j^I(i) = P_{j,0}(i)y_{j,0|0}(i) - W_0 n_{j,0|0}(i) \\ + \Lambda_1^s \left\{ \phi_j \Big[P_{j,0}(i)y_{j,1|0}(i) - W_1 n_{j,1|0}(i) \Big] + (1-\phi_j) \Big[P_{j,1}(i)y_{j,1|1}(i) - W_1 n_{j,1|1}(i) \Big] \right\}, \tag{1.2.5}$$

where $y_{j,t|\tau}(i)$ is j sector's demand for variety i at time t given the price set by the intermediate firm at time $\tau \in \{t, t-1\}$. When $\tau = t$, the demand reflects the fact that the firm is able to reset its price at period t. When $\tau = t-1$, the demand for the variety is determined by the price the firm charged the previous period (i.e. the firm can't reset the price in the current period). Additionally, $n_{j,t|\tau}(i)$ refers to labor input needed to meet the demand $y_{j,t|\tau}(i)$, as determined by the production technology (1.2.3).

Given the nominal wage rates $\{W_0, W_1\}$ and demand functions $\{y_{j,0|0}(i), y_{j,1|0}(i), y_{j,1|1}(i)\}$, an intermediate firm i in sector j chooses the optimal prices $\{P_{j,0}(i), P_{j,1}(i)\}$ and labor inputs $\{n_{j,0|0}(i), n_{j,1|0}(i), n_{j,1|1}(i)\}$ to maximize (1.2.5) given the production technology (1.2.3).

Note that conditional on being able to change the price at period $t = 0$, any two firms in sector j face an identical problem. Therefore, all resetting firms within sector j choose the same optimal price, $P_{j,t}(i) \equiv P^*_{j,t}$, $\forall\, i \in \Upsilon_{j,t}$; where $\Upsilon_{j,t} \subset [0,1]$ denotes the set of firms in sector j that are able to reset their price at period t. .

1.2.3 Borrowers and Savers

The economy is populated by two types of agents: borrowers and savers. Let $a \in \{b, s\}$ denote whether an agent is a saver (s) or a borrower (b). One of the main distinctions between these types of agents is their time preference discount parameter; $\beta_s > \beta_b$. That is, savers are assumed to be more patient than borrowers. As it will be discussed shortly, both types of agents are allowed to borrow or lend via a nominal bond. I use the terminology borrowers and savers because the more patient agents lend resources to the impatient agents through the nominal bond in the model's solution.[7] The mass of agents of type a is denoted by Ω_a, and the total population in the economy is normalized to have mass 1; hence $\Omega_s + \Omega_b = 1$.

[7]This is under the baseline calibration.

An agent of type a derives utility from consumption of the non-durable good (c^a), the durable good (d^a), and disutility from supplying work (n^a) to the intermediate firms. In supplying work, the agent is limited by a fixed endowment of H^a units of time.[8] I assume a utility function of the form $u(c,d,n) = \alpha \log(c) + (1-\alpha)\log(d) + \nu_a (1+\theta)^{-1} n^{1+\theta}$, where I set $\theta = 1$ for tractability. Therefore, the lifetime utility of an agent of type a is given by

$$U_a \equiv u(c_0, d_0, n_0) + \beta_a \Big[u(c_1, d_1, n_1) + \kappa_a \log\big((1-\delta)\, d_1 \big) \Big]. \tag{1.2.6}$$

Since the storable nature of the durable good is not well captured in the two-period horizon, I incorporate the last term in the lifetime utility (1.2.6) which is governed by the ad hoc parameter κ_a. For any model with more than two periods (think for instance of an infinite horizon model), the storability of the durable good implies that this good can potentially provide a utility flow every period after it was initially acquired. In this sense, the parameter κ_a controls the agents' incentive to store durable good at the end of the second period. Additionally, this parameter is also important for technical reasons. If the agents' incentive to store durable good in the second period is not large enough, aggregate output of the durable good at $t = 1$ might not be positive.

In order to finance the consumption of the durable and non-durable goods, agents have the following sources of income: a nominal wage rate per hour of labor supplied (W), the share of profits from ownership of the intermediate firms $\left(\widehat{\Pi}^a\right)$, and the holdings of the undepreciated durable good carried on from the previous period. Moreover, the agents have access to a nominal risk free bond (B) which can be used to transfer resources intertemporally.

Given these sources of income, the nominal budget constraint that an agent of type a faces at period $t \in \{0, 1\}$ is given by:

$$P_{c,t} c_t^a + P_{d,t} \left(d_t^a - (1-\delta)\, d_{t-1}^a \right) + R_{t-1} B_{t-1}^a = B_t^a + W_t n_t^a + \widehat{\Pi}_t^a, \tag{1.2.7}$$

where $P_{c,t}$ and $P_{d,t}$ refer to the price index of non-durable and durable goods, respectively, and δ is the depreciation rate of the durable good. The nominal rate on borrowing/lending bond contracts, agreed upon at time $t-1$, is denoted by R_{t-1}. Note that since it is assumed that labor is perfectly

[8] In the current model, it is assumed that both types of agents are identical in their total time endowment: $H^s = H^b$. Furthermore, this total time endowment is normalized to one.

mobile across the two productions sectors, the nominal wage rate is the same regardless of the sector to which the agent supplies her labor.

I assume that agents can not endogenously trade shares of their ownership of the intermediate firms. In particular, I assume that borrowers don't own shares of the intermediate firms. Savers are the sole owners and each of them has an equal share. With this in mind, let $S_j^a(i)$ denote the share of firm i in sector j owned by agent of type a, so that

$$S_j^b(i) \equiv S_j^b = 0, \quad \forall i \in [0,1]$$

$$S_j^s(i) \equiv S_j^s = \frac{1}{\Omega_s}, \ \forall i \in [0,1].$$

Hence, the share of profits from ownership of the firms for an agent of type a in period t can be written as

$$\widehat{\Pi}_t^a = \sum_{j \in \{c,d\}} \left(S_j^a \, \Pi_{j,t}^I \right). \tag{1.2.8}$$

Finally, although borrowers and savers both have access to the nominal bond market, I assume that the extent to which each of the them has access is different. While nominal lending is unrestricted for both agents (i.e. B_t^a can be as large as agent a desires); nominal borrowing is restricted. Neither type of agent can borrow in the second period ($B_1^a \geq 0$).[9] However, savers can borrow as much as they want in the initial period while borrowers face a collateral constraint of the type proposed by Kiyotaki and Moore (1997). In principle, such borrowing limit is motivated by the observation that the relatively less patient agents are more likely to default on their debt. The provision of collateral results in enough incentive for the borrowers to endogenously choose to repay the debt according to the contract. The particular form of the constraint is given by

$$R_0 B_0^b \leq (1-\chi)(1-\delta) \ d_0^b \, P_{d,1}. \tag{1.2.9}$$

For a borrower's loan amount, B_t^b, the left side of the inequality is just the repayment amount that is due at period $t = 1$. The expression to the right of the inequality sign is the collateral requirement. The agent is allowed to pledge only a fraction $(1-\chi)$ of its holdings of the non-depreciated durable

[9] This is just a technicality given the two-period nature of the model.

good as collateral. However, what is important for the provision of collateral is not the amount of the durable good that the agent holds, but its market value $d_0^b P_{d,1}$. The constraint then simply requires that the repayment value of a given loan does not exceed the collateral's market value in the repayment period.

With this specification, given initial durable good holdings $\{d_{-1}^a\}$, nominal bond holdings $\{R_{-1}B_{-1}^a\}$, prices of the consumptions goods $\{P_{c,0}, P_{d,0}, P_{c,1}, P_{d,1}\}$, nominal wage rates $\{W_0, W_1\}$, interest rate $\{R_0\}$, and intermediate firms' profits $\left\{\Pi_{c,0}^I, \Pi_{d,0}^I, \Pi_{c,1}^I, \Pi_{d,1}^I\right\}$; an agent of type a must solve the problem of choosing non-durable good consumption $\{c_0^a, c_1^a\}$, durable good consumption $\{d_0^a, d_1^a\}$, labor supplied $\{n_0^a, n_1^1\}$, and nominal bond holdings $\{B_t^a\}$, to maximize the lifetime utility (1.2.6) subject to the budget constraint (1.2.7), the time endowment $n_t^a \in [0, H^a]$, and the borrowing constraints (1.2.9) (only for agents of type borrower).

1.2.4 Monetary Authority

I assume that monetary policy is conducted via an interest rate rule of the type proposed by Taylor (1993). Given a nominal interest rate target $\tilde{R}$ and inflation target $\tilde{\pi}$, the monetary authority follows the interest rate rule given by

$$\frac{R_t}{\tilde{R}} = \left(\frac{\pi_t}{\tilde{\pi}}\right)^{\phi_\pi} z_t, \tag{1.2.10}$$

where R_t is the interest rate on nominal bond contracts and π_t is a composite inflation index that weights the inflation in the durable $\left(\pi_{d,t} \equiv \frac{P_{d,t}}{P_{d,t-1}}\right)$ and non-durable $\left(\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}\right)$ sectors according to $\pi_t = \pi_{c,t}^{\gamma}\pi_{d,t}^{1-\gamma}$. The parameter $\gamma \in (0,1)$ controls the relative weight that the monetary authority gives to each of the two sectors in the economy.[10]

Furthermore, I assume that the interest rule follows the Taylor principle with $\phi_\pi > 1$. I impose this assumption given that it is standard in the literature; where it is used for two main reasons. First, the empirical evidence suggests that the failure of the monetary authority to follow this principle has led to episodes of greater macroeconomic instability in the US.[11] Second, the Taylor principle usually arises as a necessary and sufficient condition for the existence of a unique stable solution in

[10] For the baseline model, γ is set to 0.5 so that the monetary authority weights the two sectors equally.
[11] See Taylor (1999) and Clarida, Galí and Gertler (2000).

a class of infinite horizon New Keynesian forward looking models.[12]

The component z_t captures some of the exogenous factors to which the monetary authority might react. In my model, this component is deterministic and follows the process

$$\log(z_0) = \epsilon_0^z$$

$$\log(z_1) = \rho \log(z_0)$$

Note that ϵ_0^z is known to the agents and firms in the initial period when they make their optimal decisions.

Finally, given the two-period nature of the model, the nominal interest rate in the last period (R_1) plays no role in the agents' decisions. Therefore, I assume it is fixed and set $R_1 = 1$ for convenience.

1.3 Solution

Since the goal of the chapter is to understand how credit constraints affect the equilibrium outcome within this New Keynesian framework, I consider two versions of the model outlined in Section 1.2. The first version refers to the economy exactly as described in Section 1.2 and is denoted by $E = C$; where C stands for constrained. The second version just assumes that the borrowers *do not face credit constraints* and is denoted by $E = NC$; where NC stands for non-constrained.

1.3.1 Agents' Optimization Problems

Let $E \in \{\text{NC}, \text{C}\}$ denote whether we are considering an economy without credit constraints (NC) or an economy with credit constraints (C), $a \in \{s, b\}$ denote an agent's type, $j \in \{c, d\}$ denote a firm's sector, and $t \in \{0, 1\}$ denote the period.

[12] See Woodford (2001) and Woodford (2003).

For an agent of type a the problem can be formulated as

$$\max_{\{c_t^a, b_t^a, d_t^a, n_t^a\}_{t=0}^1} u(c_0^a, d_0^a, n_0^a) + \beta_a \Big[u(c_1^a, d_1^a, n_1^a) + \kappa_a \log\big((1-\delta)\, d_1^a\big)\Big] \tag{1.3.1}$$

$$s.t. \quad c_t^a + q_t\left(d_t^a - (1-\delta)\, d_{t-1}^a\right) = b_t^a + \omega_{c,t} n_t^a - R_{t-1} b_{t-1}^a \pi_{c,t}^{-1} + \Pi_t^a$$

$$R_0 b_0^b \leq (1-\chi)(1-\delta)\, q_0 \pi_{d,1} d_0^b, \quad \text{if } E = C$$

$$c_t^a, d_t^a, n_t^a, b_1^a \geq 0$$

where the budget and credit constraints are written in real terms. Therefore, $b_t \equiv \frac{B_t}{P_{c,t}}$ denotes bond holdings in non-durable units, $\omega_{c,t} \equiv \frac{W_t}{P_{c,t}}$ is the real wage rate in non-durable units, $\Pi_t \equiv \frac{\widehat{\Pi}_t}{P_{c,t}}$ are real profits in non-durable units, and $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$ is the relative price of durables.

Similarly, for an intermediate firm, the problem can be written as

$$\text{Period } t=0: \quad \Pi_{j,0} = \max_{\hat{p}_{j,0} \geq 0} \left\{ Y_{j,0} P_{j,0} \left(\hat{p}_{j,0}\right)^{-\epsilon_j} \left[\left(\hat{p}_{j,0} - \omega_{j,0}\right) + \beta_s \phi_j D_j \pi_{j,1}^{\epsilon_j} \pi_{c,1}^{-1} \left(\hat{p}_{j,0} - \omega_{j,1} \pi_{j,1}\right) \right] \right\} \tag{1.3.2}$$

$$\text{Period } t=1: \quad \Pi_{j,1} = \max_{\hat{p}_{j,1} \geq 0} \left\{ Y_{j,1} P_{j,1} \left(\hat{p}_{j,1}\right)^{-\epsilon_j} \left(\hat{p}_{j,1} - \omega_{j,1}\right) \right\}$$

where $\hat{p}_{j,t} \equiv \frac{P_{j,t}^*}{P_{j,t}}$ is the relative price chosen by the resetting firms, $\omega_{j,t} \equiv \frac{W_t}{P_{j,t}}$ is the real wage rate in units of j sector's good, and $D_j \equiv \frac{c_0^s}{c_1^s} \left(\frac{Y_{j,0}}{Y_{j,1}}\right)^{-1}$ is a factor that captures the firm's intertemporal discounting.

For both of the economies $E \in \{NC, C\}$, the solutions to the problems (1.3.1) and (1.3.2) are completely characterized by the first order conditions. To see this, note that the utility function $u(c,d,n)$ is concave and satisfies the Inada conditions $\lim_{c \to 0} u_c(c,d,n) = \infty$ and $\lim_{d \to 0} u_d(c,d,n) = \infty$. Additionally, the constraints are continuously differentiable convex functions, so that the first order conditions (FOC's) are necessary and sufficient to characterize an interior solution to (1.3.1). Similarly, it can be easily seen that the objective functions in (1.3.2) are strictly concave in $\hat{p}_{j,t}$ and the only constraint is the non-negativity constraint. Hence, the FOC's associated with these problems are also necessary and sufficient to characterize the solution.

1.3.2 Aggregate Variables

In addition to the aggregate sectorial output ($Y_{j,t}$), price index ($P_{j,t}$), and inflation ($\pi_{j,t}$); one can define two additional aggregate variables: sectorial labor employed ($N_{j,t}$) and sectorial price dispersion ($\Delta_{j,t}$).

For each period $t \in \{0,1\}$, aggregate labor employed in sector j is given by

$$N_{j,t} \equiv \int_0^1 n_{j,t}(i)\, di$$
$$N_{j,t} = Y_{j,t} \int_0^1 \left(\frac{P_{j,t}}{P_{j,t}(i)} \right)^{\epsilon_j} di, \tag{1.3.3}$$

where the second expression follows given an intermediate firm's production function and the final producer's demand for variety $y_{j,t}(i)$. Equation (1.3.3) states that one can think of a representative firm in each sector. This representative firm produces the final sectorial good using aggregate labor according to a production function of the form

$$Y_{j,t} = \Delta_{j,t}^{-1} \cdot N_{j,t},$$

where

$$\Delta_{j,t} \equiv \int_0^1 \left(\frac{P_{j,t}}{P_{j,t}(i)} \right)^{\epsilon_j} di. \tag{1.3.4}$$

Note that $\Delta_{j,t}$ is a measure of sectorial price dispersion; it captures the extent to which the price of each intermediate variety i deviates from the sectorial price index. For the remainder of the paper, equation (1.3.3) will be called the aggregate production function and (1.3.4) the sectorial price dispersion.

In light of the price adjustment friction in the intermediate sectors, the inflation index and price dispersion can be written as

$$\Delta_{P_{j,t}} = \int_{\Upsilon_{j,t}} \left(\frac{P_{j,t}}{P_{j,t}(i)} \right)^{\epsilon_j} di + \int_{[0,1]\setminus\Upsilon_{j,t}} \left(\frac{P_{j,t}}{P_{j,t-1}(i)} \right)^{\epsilon_j} di$$

$$\pi_{j,t}^{1-\epsilon_j} = \int_{\Upsilon_{j,t}} \left(\frac{P_{j,t}(i)}{P_{j,t-1}} \right)^{1-\epsilon_j} di + \int_{[0,1]\setminus\Upsilon_{j,t}} \left(\frac{P_{j,t-1}(i)}{P_{j,t-1}} \right)^{1-\epsilon_j} di;$$

where $\Upsilon_{j,t} \subset [0,1]$ refers to the set of resetting firms. These two expressions summarize the aggregate price dynamics of the economy. They relate the relevant aggregate pricing variables in the current period to the distribution of intermediate firms' prices in the previous period.

Given the i.i.d. nature of the stochastic price resetting process and the observation that all resetting firms face the same problem so that $P_{j,t}(i) \equiv P_{j,t}^*$, $\forall\, i \in \Upsilon_{j,t}$, the previous expressions can be manipulated to obtain the following

$$\pi_{j,t}^{1-\epsilon_j} = (1-\phi_j) \left(\frac{\hat{p}_{j,t}}{\pi_{j,t}} \right)^{1-\epsilon_j} + \phi_j \tag{1.3.5}$$

$$\Delta_{j,t} = \left[(1-\phi_j)^{-1} \left(1 - \phi_j \pi_{j,t}^{\epsilon_j - 1} \right)^{\epsilon_j} \right]^{1/(\epsilon_j - 1)} + \phi_j \pi_{j,t}^{\epsilon_j} \Delta_{j,t-1}, \tag{1.3.6}$$

where $\hat{p}_{j,t} \equiv \frac{P_{j,t}^*}{P_{j,t}}$. Note that although the set of resetting firms is random, the measure of such set is deterministic and equal to $1-\phi_j$. With this in mind, equation (1.3.5) just states that a measure ϕ_j of firms doesn't contribute to inflation (as they are unable to reset their price). The remaining measure $1-\phi_j$ contributes to inflation whenever their adjusted price differs from the previous period price index. Equation (1.3.6) summarizes the price dispersion dynamics for the economy. The current period price dispersion depends on two additively separable factors. First, on the price chosen by the firms that can change their price. Second, for the firms that are unable to reset their price, their contribution to the current price dispersion depends on the previous period price dispersion.

Aggregate nominal profits for the intermediate firms in sector j can be compactly written as

$$\Pi_{j,t}^I = \int_0^1 \Big(P_{j,t}(i) y_{j,t}(i) - W_t n_{j,t}(i) \Big) di = P_{j,t} Y_{j,t} - W_t N_{j,t}. \tag{1.3.7}$$

Again, in terms of profits, one can think of a representative firm in each sector which inherits the linear production technology from the individual firms. It produces the final good and sells it at the aggregate price; the only input for production is aggregate labor, which must be compensated by the nominal wage rate.

1.3.3 Equilibrium Concept

Consider the economy $E \in \{NC, C\}$. Given the initial monetary policy response to exogenous events (ϵ_0^z), the initial asset holdings $\left(R_{-1}b_{-1}^a, d_{-1}^a\right)_{a\in\{s,d\}}$ and prices $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$, the equilibrium is given by sequences

$$(i)\text{ Agents' allocations: } \mathbf{A^E} \equiv \left\{\left\{c_{a,t}^{\text{E}}, d_{a,t}^{\text{E}}, n_{a,t}^{\text{E}}, b_{a,0}^{\text{E}}\right\}_{a\in\{s,b\}}\right\}_{t=0}^{1}$$

$$(ii)\text{ Aggregate sectorial output and labor: } \mathbf{G^E} \equiv \left\{Y_{c,t}^{\text{E}}, Y_{d,t}^{\text{E}}, N_{c,t}^{\text{E}}, N_{d,t}^{\text{E}}\right\}_{t=0}^{1}$$

$$(iii)\text{ Sectorial profits and prices: } \mathbf{P^E} \equiv \left\{\left\{\Pi_{j,t}^{\text{E}}, \pi_{j,t}^{\text{E}}, \Delta_{j,t}^{\text{E}}, \omega_{j,t}^{\text{E}}, q_t^{\text{E}}, R_0^{\text{E}}\right\}_{j\in\{c,d\}}\right\}_{t=0}^{1};$$

such that:

1. Agents Optimize: Given $\mathbf{P^E}$ and $\left(R_{-1}b_{-1}^a, d_{-1}^a\right)$, $\mathbf{A^E}$ satisfies the FOC's of (1.3.1) for $a \in \{s, b\}$.
2. Intermediate Firms Optimize: Given $\mathbf{G^E}$ and $\mathbf{P^E}$, $\hat{p}_{j,t}$ as defined by (1.3.5) satisfies the FOC's of (1.3.2) for $j \in \{c, d\}$.
3. Monetary Policy: Given $\left(\epsilon_z^0\right)$, $\mathbf{P^E}$ satisfies (1.2.10).
4. Law of Motion of Aggregate Prices: Given $(\Delta_{c,-1}, \Delta_{d,-1})$, $\mathbf{P^E}$ satisfies (1.3.6).
5. Debt Market clears: $\Omega_s b_{s,0}^{\text{E}} + \Omega_b b_{b,0}^{\text{E}} = 0$
6. Labor Market clears for $t \in \{0, 1\}$: $\Omega_s n_{s,t}^{\text{E}} + \Omega_b n_{b,t}^{\text{E}} = N_{c,t}^{\text{E}} + N_{d,t}^{\text{E}}$
7. Non-durable market clears for $t \in \{0, 1\}$: $Y_{c,t} = \Omega_s c_t^s + \Omega_b c_t^b$
8. Durable market clears for $t \in \{0, 1\}$: $Y_{d,t} = \Omega_s \left(d_t^s - (1-\delta)\, d_{t-1}^s\right) + \Omega_b \left(d_t^b - (1-\delta)\, d_{t-1}^b\right)$

1.4 Results

The following section presents the main results of the chapter. I start by summarizing the partial equilibrium results of introducing the credit constraint in a series of theoretical propositions. Next, I conduct a numerical exercise to illustrate these partial equilibrium results and to provide intuition for the general equilibrium results.[13]

1.4.1 Partial Equilibrium: A Theoretical Analysis

The analysis that follows focuses on understanding how the equilibrium sequences $\mathbf{A^E}$, $\mathbf{G^E}$, and $\mathbf{P^E}$ change when going from an economy without credit constraints ($E = NC$) to one with credit constraints ($E = C$). In order to have a meaningful comparison, one needs to ensure the existence and uniqueness of a solution to the agents' problems in each one of the two economies.

Proposition 1.1 *Consider the economy without credit constraints* $(E = NC)$ *for an agent of type* $a \in \{s, b\}$*. Let the initial wealth holdings be given by* $\left(R_{-1}b^a_{-1}, d^a_{-1}\right)$*. Suppose the prices* $(\pi_{j,t}, q_{j,t}, \Delta_{j,t}, \omega_{j,t}, R_0)$ *are such that* $R_0 - (1-\delta)\,\pi_{d,1} \geq 0$*. Then a unique interior solution to* (1.3.1) *exists which is consistent with the definition of* Π^a_t*.*

Proposition 1.1 states that as long as the return on real debt R_0 is large enough relative to the return on the stored durable good $(1-\delta)\,\pi_{d,0}$, a unique interior solution to the borrower's / saver's problem exists. Intuitively, if the return on real debt is not large enough, the agents would rather store wealth in the form of durable good. However, to the extent that the durable good is also a substitute for the non-durable good, a large durable consumption can potentially lead to zero non-durable consumption.

Proposition 1.2 *Consider the economy with credit constraints* $(E = C)$ *for an agent of type* $a \in \{s, b\}$*. Suppose that the assumptions of Proposition 1.1 hold. Then an interior solution to* (1.3.1) *exists which is consistent with the definition of* Π^a_t*. If the borrowing constraint is non-binding, then this solution is unique and it is the same one as for the economy without credit constraints.*

For the solution where the borrowing constraint is binding, define

(a) $P_1 \equiv \beta_b\,(1 + (1-\alpha)\,\kappa_b)$,

[13] The proofs of the propositions are presented in Appendix A.1.

(b) $W^a_{-1} \equiv q_0 (1-\delta) d^a_{-1} - R_{-1} b^a_{-1} \pi^{-1}_{c,0}$,

(c) $A_1 \equiv \chi^{-1} \left(R_0 (1-\delta)^{-1} \pi^{-1}_{d,1} - 1 \right)$,

(d) $A_2 \equiv \beta_b^{-1} \left(\omega_{c,1} \pi^{-1}_{c,1} R_0^{-1} \right)^2 (1 + A_1)^2$,

(e) $A_3 \equiv 4\, \nu_b^{-1} \left(\omega^2_{c,0} + A_2 \right) (1 + P_1)$.

If $P_1 > 1$*,* $W^b_{-1} \geq 0$*, and* $\left(W^b_{-1}\right)^2 \geq \max \left\{ A_3 - 8 \nu_b^{-1} \omega^2_{c,0} (1 + P_1),\ A_3 \left(P_1^2 - 1 \right)^{-1} \right\}$*; then this binding-solution is unique.*

Proposition 1.2 is the equivalent of Proposition 1.1 for the economy with credit constraints. Again, as long as the return on real debt is large enough compared to the return on durable goods, an interior solution exists where agents consume positive amounts of the durable and non-durable goods.

However, there are two additional sufficient conditions required for the uniqueness of the borrower's solution when the credit constraints are binding. First, the storable nature of the durable good must be sufficiently attractive for the borrower; the condition $P_1 > 1$ implies that κ_b must be sufficiently large. Second, the borrower's initial wealth W^b_{-1} must be large enough to allow positive consumption of the durable and non-durable goods given the borrowing constraint.

As it is clear from Proposition 1.2, the equilibrium allocation in the economy with credit constraints is the same as that for the economy without credit constraints whenever the borrowing constraint is non-binding. Therefore, in the analysis that follows, the only non-trivial comparison between the two economies arises for the case of a binding borrowing constraint. The next proposition establishes the conditions under which this is the case.

Proposition 1.3 *Suppose that the conditions of Propositions 1.1 and 1.2 hold. Let* $\widehat{\boldsymbol{A}}^{\boldsymbol{NC}}$*,* $\widehat{\boldsymbol{G}}^{\boldsymbol{NC}}$*, and* $\widehat{\boldsymbol{P}}^{\boldsymbol{NC}}$ *denote the equilibrium agents' allocation, aggregate output, and price sequences for an economy without credit constraints.*

Define W^b_{-1}*,* P_1*, and* A_1 *as in Proposition 1.2 and*

(a) $B_1 \equiv \beta_b^{-1} \left(\hat{\omega}^{NC}_{c,1} \hat{\pi}^{NC}_{c,1} \left(\hat{R}^{NC}_0 \right)^{-1} \right)^2$,

(b) $B_2 \equiv 4 \nu_b^{-1} \left({\hat{\omega}^{NC}_{c,0}}{}^2 + B_1 \right) (1 + P_1)$,

(c) $B_3 \equiv 4 \nu_b^{-1} A_1 \left(A_1 P_1 - (1 - \alpha) \right)^{-1} B_1 (1 + P_1)^2$.

Then $\widehat{\boldsymbol{A}}^{\boldsymbol{C}} \neq \widehat{\boldsymbol{A}}^{\boldsymbol{NC}}$, $\widehat{\boldsymbol{G}}^{\boldsymbol{C}} \neq \widehat{\boldsymbol{G}}^{\boldsymbol{NC}}$, *and* $\widehat{\boldsymbol{P}}^{\boldsymbol{C}} \neq \widehat{\boldsymbol{P}}^{\boldsymbol{NC}}$, *if and only if* $W_{-1}^{b} \leq (B_3 - B_2)\left(2\sqrt{B_3}\right)^{-1}$ *whenever* $A_1 P_1 > (1-\alpha)$.

Proposition 1.3 provides conditions for which the equilibrium of the economy with credit constraints is different from that of the economy without credit constraints. From the previous discussion it is clear that the two equilibriums are different as long as the credit constraint is binding. Hence, an equivalent interpretation of Proposition 1.3 is that it provides conditions under which the credit constraint is binding. To understand the intuition underlying this proposition, note that the expression $A_1 P_1 > (1-\alpha)$ is equivalent to:

$$(1-\alpha)\, P_1 \left[\frac{(1-\chi) q_0 d_0^b}{b_0^b} - 1 \right] > (1-\alpha)\, \chi,$$

where I have used the fact that $\dfrac{(1-\chi) q_0 d_0^b}{b_0^b} = \dfrac{R_0}{(1-\delta)\pi_{d,1}} \geq 1$ (the last inequality follows from the assumption in Proposition 1.1). The left hand side of the inequality can be interpreted as a marginal cost of holding real debt given the binding constraint. The agent is now forced to hold some durable good in the form of collateral. The term in brackets captures the value of collateral durable holdings required per unit of real debt. In the second period, the agent uses this collateral to repay the debt instead of consuming it to derive utility. The factor $(1-\alpha)P_1$ captures the discounted marginal utility of durable consumption in the second period and its continuation value. Thus this term represents the *foregone* marginal utility due to the binding constraint. The right hand side is the marginal benefit of holding debt. For every additional unit of debt, the agent must now increase her holdings of durable good, but only the fraction $(1-\chi)$ can be pledged as collateral while the remaining fraction χ is consumed. The right hand side term captures the marginal utility associated with this consumption.

Thus Proposition 1.3 states that, whenever the marginal cost of holding debt is larger than its marginal benefit, the agent's initial wealth holdings must be sufficiently low for the constraint to be binding. The low initial wealth holdings provide enough incentive for the agent to acquire debt to the point where the constraint becomes binding. When the marginal cost of holding debt is at most equal to its marginal benefit, $A_1 P_1 \leq (1-\alpha)$, the agent will acquire debt to the point where the constraint is binding *regardless* of her initial wealth holdings.

To understand the effect that the binding constraint has on the economy's equilibrium, I conduct the following thought experiment. Consider the economy without binding constraints, whose equilibrium is given by the allocation $\widehat{\mathbf{A}}^{\mathbf{NC}}$, aggregate output $\widehat{\mathbf{G}}^{\mathbf{NC}}$, and prices $\widehat{\mathbf{P}}^{\mathbf{NC}}$. Suppose now that the credit constraint for the borrowers is introduced. If the conditions of Proposition 1.3 hold, we know $\left\{\widehat{\mathbf{A}}^{\mathbf{NC}}, \widehat{\mathbf{G}}^{\mathbf{NC}}, \widehat{\mathbf{P}}^{\mathbf{NC}}\right\}$ is not an equilibrium of this economy. To see how this equilibrium changes, I proceed in two steps. First, I consider the new *partial equilibrium* of the economy with credit constraints given the aggregate variables $\widehat{\mathbf{G}}^{\mathbf{NC}}$ and prices $\widehat{\mathbf{P}}^{\mathbf{NC}}$. That is, I focus on how the borrowers' allocation changes once the constraint is introduced assuming the prices and aggregate output remain fixed. I denote this partial equilibrium allocation by $\widetilde{\mathbf{A}}^{\mathbf{C}}$. Next, I allow for general equilibrium effects. That is, starting with the allocation $\widetilde{\mathbf{A}}^{\mathbf{C}}$, I study how it changes as prices adjust in order for markets to clear. The resulting sequence $\left\{\widehat{\mathbf{A}}^{\mathbf{C}}, \widehat{\mathbf{G}}^{\mathbf{C}}, \widehat{\mathbf{P}}^{\mathbf{C}}\right\}$ corresponds to the equilibrium of the economy with (binding) credit constraints.

Proposition 1.4 summarizes the result from the first step in the procedure; the partial equilibrium effects on the borrowers allocation when the credit constraint is introduced.

Proposition 1.4 *Given $\widehat{\boldsymbol{P}}^{NC}$, suppose $\left(\hat{c}_{b,t}^{NC}, \hat{d}_{b,t}^{NC}, \hat{n}_{b,t}^{NC}, \hat{b}_{b,0}^{NC}\right) \in \widehat{\boldsymbol{A}}^{NC} \in$ is the borrower's equilibrium allocation. Let $\left(\tilde{c}_{b,t}, \tilde{d}_{b,t}, \tilde{n}_{b,t}, \tilde{b}_{b,0}\right) \in \widetilde{\boldsymbol{A}}^{C}$ denote the corresponding borrower's partial equilibrium allocation in the economy with the binding constraint. Suppose that the assumptions of Propositions 1.1 and 1.2 hold. Then*

(a) $\tilde{d}_{b,0} \leq d_{b,0}^{NC}$, $\tilde{b}_{b,0} \leq b_{b,0}^{NC}$.

(b) $\tilde{c}_{b,0} \leq c_{b,0}^{NC}$, $\tilde{n}_{b,0} \geq n_{b,0}^{NC}$

(c) $\tilde{c}_{b,1} \geq c_{b,1}^{NC}$, $\tilde{d}_{b,1} \geq d_{b,1}^{NC}$, $\tilde{n}_{b,1} \leq n_{b,1}^{NC}$

The intuition behind Proposition 1.4 boils down to recognizing that the introduction of the credit constraint has two effects; a wealth effect and a substitution effect.

Consider first the wealth effect on consumption. Given the prices and the binding credit constraint, for any level of durable good the borrower has access to less debt. Relative to the economy without credit constraints, this implies that the borrower's non-labor wealth in the initial period must be smaller while the final period non-labor wealth must be larger. As consumption of the durable and non-durable goods is an increasing function of non-labor wealth, this would tend to decrease overall consumption in the initial period while increasing it in the final period. This is the wealth effect.

The substitution effect is a consequence of the restriction that a fraction $\chi \in (0,1)$ of the durable good cannot be pledged as collateral. Given that only a fraction of the durable good can be collateralized, the relative price of the durable good is larger when the constraint is binding. For the economy without credit constraints, the effective relative price of the durable good is given by $q_{0,\text{eff}}^{\text{NC}} \equiv q_0 \left(1 - (1-\delta)\, \pi_{d,1} R_0^{-1}\right)$. For every unit of the durable, the borrower is giving up less than q_0 units of the non-durable. This is because the borrower can always sell the durable, net of depreciation, at its market price in the next period an then transfer that wealth back to the current period, via debt, and use it to buy non-durable. This is feasible since there is no financial frictions and markets are complete. For the economy with a binding credit constraint, the effective relative price of the durable good is now $q_{0,\text{eff}}^{\text{C}} \equiv q_0 \left(1 - (1-\chi)(1-\delta)\, \pi_{d,1} R_0^{-1}\right)$. Again, for every unit of durable the borrower gives up less than q_0 units of the non-durable. However, when the borrower tries to use debt to transfer the wealth gained by selling the non-depreciated durable in the next period, he faces the constraint that for every unit of durable, he can only transfer back the fraction $(1-\chi)$. Hence, when the constraint is binding, the borrower has an incentive to substitute durable with non-durable consumption in the initial period as the durable good is now relatively more costly.

Thus the results in Proposition 1.4 (a) follow immediately. The wealth and substitution effects imply consumption of the durable good in the initial period must decrease; it is both less affordable and less attractive. In turn, this implies that the debt holdings also decrease as the constraint is binding and less of the durable good is consumed. Note that these results are independent of the functional form of the utility and they rely only on the assumption of monotonicity and concavity.

The remaining results (b) and (c) in Proposition 1.4 depend on the particular functional form of the utility. For non-durable consumption and labor supplied in the initial period, the wealth and substitution effects operate in opposite directions. Thus whether they increase or decrease depends on which one of these two effects dominates. For the assumed log-linear and separable utility function, the wealth effect is always dominant. Similarly, for the second period variables, the wealth effect dominates so that the borrower is able to consume more of both, durable and non-durable goods while working less hours.

Having characterized how the borrower's partial equilibrium allocation changes due to the binding credit constraint, I now turn to the general equilibrium effects. Propositions 1.5 and 1.6 introduce two results that make it simpler to characterize the equilibrium prices and aggregate output for the economy $E \in \{C, NC\}$.

Proposition 1.5 *For the economy E and given the price vector* $\boldsymbol{P^E}$ *satisfying* $\omega_{c,t}, R_0, \pi_{c,t}^{-1} > 0$, *consider an allocation* $\boldsymbol{A}^E$ *that solves the borrower's and saver's problems as given by* (1.3.1). *Let* $\Omega_b R_{-1} b_{-1}^b + \Omega_s R_{-1} b_{-1}^s = 0$. *Then the following three statements are equivalent:*

(a) $\Omega_s n_0^s + \Omega_b n_0^b = N_{c,0} + N_{d,0}$

(b) $\Omega_s b_0^s + \Omega_b b_0^b = 0$

(c) $\Omega_s n_1^s + \Omega_b n_1^b = N_{c,1} + N_{d,1}$

Proposition 1.5 states that if an allocation solves the agents' problems, then it is an equilibrium allocation if and only if it clears one of three markets: labor market in the initial period, labor market in the final period, or debt market in the initial period. In terms of computing the equilibrium for the economy E, this proposition is useful as it implies one only needs to ensure that *one* of these three markets clears and then the other two will automatically clear. Proposition 1.6 takes advantage of this result to characterize the equilibrium of the economy E as a function of only *one* element of the price vector $\mathbf{P}^E$.

Proposition 1.6 *Fix the initial state of the economy E given by* z_0, $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$, *and* $\left(R_{-1} b_{-1}^a, d_{-1}^a\right)_{a \in \{s,b\}}$. *Then*

(a) $\boldsymbol{A}^E = \boldsymbol{A}^E\left(\omega_{c,0}\right)$

(b) $\boldsymbol{G}^E = \boldsymbol{G}^E\left(\omega_{c,0}\right)$

(c) $\boldsymbol{P}^E = \boldsymbol{P}^E\left(\omega_{c,0}\right)$

Thus, in computing the equilibrium for the economy E, all the relevant variables can be written as a functions of only the initial period's real wage rate. In other words, computing the equilibrium boils down to finding the real wage rate that clears the labor market in the initial period.

It is clear that to understand the effect of the credit constraint, it suffices to focus on its effect on the equilibrium real wage rate. To do so, I proceed in two steps. First, Proposition 1.7 establishes the effect of the credit constraint on the initial period's labor market. Second, Proposition 1.8 establishes how the real wage rate must adjust to restore this labor market to equilibrium.

Proposition 1.7 *Fix the initial state of the economy given by* z_0*,* $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$*, and* $\left(R_{-1}b^a_{-1}, d^a_{-1}\right)_{a\in\{s,b\}}$*. Consider the equilibrium price vector* $\widehat{\boldsymbol{P}}^{\boldsymbol{NC}}$ *for the economy without credit constraints. Let* $\tilde{c}_{b,0}$ *be the optimal consumption of an agent of type borrower in the economy with credit constraints given the prices* $\widehat{\boldsymbol{P}}^{\boldsymbol{NC}}$ *and denote by* $\tilde{\zeta}_{b,0} \geq 0$ *the corresponding credit constraint multiplier.*

Define:

(a) $Q_0 \equiv \alpha^{-1}(1-\alpha)\hat{R}^{NC}_0\left(\hat{R}^{NC}_0 - (1-\delta)\hat{\pi}^{NC}_{d,1}\right)^{-1}$

(b) $Q^a_1 \equiv \alpha^{-1}(1-\alpha)\left[\beta_a(1+\kappa_a) - (1-\delta)\hat{\pi}^{NC}_{d,1}\left(\hat{R}^{NC}_0 - (1-\delta)\hat{\pi}^{NC}_{d,1}\right)^{-1}\right]$*, for* $a \in \{s,b\}$

(c) $\overline{W}^s_{-1} \equiv \Omega^{-1}_s(1-\delta)\left[\hat{\Delta}^{NC}_{d,0}\hat{\omega}^{NC}_{c,0}\left(\Omega_s d^s_{-1} + \Omega_b d^b_{-1}\right) - \hat{q}^{NC}_0\Omega_b d^b_{-1}\right] - R_{-1}b^s_{-1}\left(\hat{\pi}^{NC}_{c,0}\right)^{-1}$

(d) $K^a_R \equiv 1 + Q_0 + \beta_a + Q^a_1$*, for* $a \in \{s,b\}$

(e) $K^a_C \equiv \hat{\omega}^{NC}_{c,0}\left[\hat{\Delta}^{NC}_{c,0} + \left(\hat{q}^{NC}_0\right)^{-1}\hat{\Delta}^{NC}_{d,0}Q_0\right] + \hat{\omega}^{NC}_{c,1}\left[\hat{\Delta}^{NC}_{c,1}\beta_a + \left(\hat{q}^{NC}_1\right)^{-1}\hat{\Delta}^{NC}_{d,1}Q^a_1\right]$*, for* $a \in \{s,b\}$

(f) $A_{c,t} \equiv \left(1 - \hat{\omega}^{NC}_{c,t}\hat{\Delta}^{NC}_{c,t}\right)$

(g) $A_{d,t} \equiv \left(\hat{q}^{NC}_t - \hat{\omega}^{NC}_{c,t}\hat{\Delta}^{NC}_{d,t}\right)$

(h) $B_\zeta \equiv \alpha^{-1}\hat{R}^{NC}_0\tilde{c}_{b,0}\tilde{\zeta}_{b,0}$

(i) $D_\zeta \equiv \alpha^{-1}\chi(1-\delta)\hat{\pi}^{NC}_{d,1}\left(\hat{R}^{NC}_0 - (1-\delta)\hat{\pi}^{NC}_{d,1}\right)^{-1}\tilde{c}_{b,0}\tilde{\zeta}_{b,0}$

(j) $\Delta_{K,d_0} \equiv (1-D_\zeta)^{-1}D_\zeta\left(\hat{q}^{NC}_0\right)^{-1}Q_0\left(A_{d,0} - \left(\hat{R}^{NC}_0\right)^{-1}\hat{\pi}^{NC}_{c,1}(1-\delta)A_{d,1}\right)$

(k) $\Delta_{K,c_1} \equiv (1-B_\zeta)^{-1}B_\zeta\,\beta_b\left[A_{c,1} + \left(\alpha\,\hat{q}^{NC}_1\right)^{-1}(1-\alpha)(1+\kappa_b)A_{d,1}\right]$

(l) $\Delta_K \equiv \Delta_{K,d_0} + \Delta_{K,c_1}$

Finally, let $N_0^C(\omega_{c,0}) \equiv N_{c,0}^C(\omega_{c,0}) + N_{d,0}^C(\omega_{c,0}) - \left[\Omega_s n_{s,0}^C(\omega_{c,0}) + \Omega_b n_{b,0}^C(\omega_{c,0})\right]$ *denote the initial period's excess labor demand for the economy with credit constraints as a function of the real wage rate.*

Then

1. *Case 1: If* $\hat{c}_{b,0}^{NC} \geq \tilde{c}_{b,0}\left(K_R^b - K_C^b\right)^{-1}\left(K_R^b - K_C^b + \Delta_K\right) \implies N_0^C\left(\hat{\omega}_{c,0}^{NC}\right) \leq 0.$
2. *Case 2: If* $\hat{c}_{b,0}^{NC} < \tilde{c}_{b,0}\left(K_R^b - K_C^b\right)^{-1}\left(K_R^b - K_C^b + \Delta_K\right) \implies N_0^C\left(\hat{\omega}_{c,0}^{NC}\right) \leq 0$ *whenever* κ_s *large enough and* $\overline{W}_{-1}^s \geq \Omega_s^{-1}\hat{c}_{b,0}^{NC}\left[\Omega_s K_C^s + \Omega_b\left(K_C^b - K_R^b\right)\right]$.

Proposition 1.7 states that, under certain conditions, the introduction of the credit constraint results in excess labor supplied at the equilibrium wage rate of the economy without credit constraints, $\hat{\omega}_{c,0}^{\text{NC}}$. The particular conditions are related to how the intermediate firms' profits change once the credit constraint is introduced. As the borrowers adjust their allocation, the intermediate firms must adjust their production to meet demand, thus affecting profits. Given the assumption that the intermediate firms are solely owned by the savers, the change in profits induces a wealth effect which results in a change in the savers' allocation. The extent to which the savers adjust their allocation depends on the sign and size of the change in profits. The conditions of Proposition 1.7 ensure that there is excess labor supplied even in the event in which the profit change induces savers to decrease their labor supplied.

To understand the nature of the conditions imposed in Proposition 1.7, note first that the change in profits has two components: the intensive and extensive margins. The intensive margin refers to the change in profits *per unit* of non-durable consumption. The extensive margin alludes to the change in profits due to the total change of non-durable consumption by the borrower. The net effect on profits is determined by the combination of these two margins.

In terms of the intensive margin, an intermediate firm's real profits for selling *one unit of good* $j \in \{c, d\}$ *at period* t are given by $A_{j,t}$, where $\omega_{c,t}\Delta_{j,t}$ denotes the real unit cost of good j. Now, think of an economy without credit constraints where an agent of type a consumes the *optimal* amounts of c_0^a, d_0^a, c_1^a, and d_1^a. That is, the borrower consumes $R_0\pi_{c,1}^{-1}\beta_b$ units of c_1^b and $q_0^{-1}Q_0$ units of d_0^b per unit of c_0^b. Given this optimal behavior, for every unit of c_0^a the agent consumes, the non-durable sector makes a real revenue of 1 and β_b in each period while the durable sector makes a real revenue of Q_0 and Q_1^a. Therefore, K_R^a refers to the revenue (per unit of non-durable) generated by the intermediate firms when an agent of type a behaves optimally. Similarly, K_C^a refers to the firms' cost (per unit of non-durable) when supplying the optimal allocation to an agent of type a. That is, for every unit of non-durable that an agent consumes, the intermediate firms' total real profits in the economy without credit constraints are given by $T_a^{\mathrm{NC}} \equiv K_R^a - K_C^a$.

Consider next an economy with credit constraints where, again, an agent of type a behaves optimally. For the agents of type borrower, the constraint introduces new tradeoffs in their consumption decision. The variables $1 \geq D_\zeta,\ B_\zeta \geq 0$ capture these new tradeoffs. With the constraint, the borrower consumes $(1 - D_\zeta)^{-1} R_0 \pi_{c,1}^{-1} \beta_b$ units of c_1^b and $(1 - B_\zeta)^{-1} q_0^{-1} Q_0$ units of d_0^b per unit of c_0^b; the constraint increases the borrowers per unit consumption of both goods. Given this change in consumption, Δ_{K,c_1} and Δ_{K,d_0} denote the change in per unit profits in each sector when agents behave optimally. It is clear that $\Delta_{K,c_1} \geq 0$ but Δ_{K,d_0} might be positive or negative, depending on how borrowers reallocate durable consumption between the initial and final periods when the constraint is introduced. Thus, for every unit of non-durable that the borrower consumes, the intermediate firms' total real profits in the economy with credit constraints are given by $T_b^{\mathrm{C}} \equiv K_R^b - K_C^b + \Delta_K$.[14]

While the intensive margin might result in an increase or decrease in profits per unit of non-durable consumption, the extensive margin is always negative. From Proposition 1.4, introducing the constraint decreases non-durable consumption for the borrowers, $\tilde{c}_{b,0} \leq \hat{c}_{b,0}^{\mathrm{NC}}$, thus implying a decrease in profits for the intermediate firms. The total effect on profits is given by the combination of these two margins. For the economy without credit constraints, the total real profits are $T^{\mathrm{NC}} \cdot \hat{c}_{b,0}^{\mathrm{NC}}$; while for the economy with credit constraints they are $T_b^{\mathrm{C}} \cdot \tilde{c}_{b,0}$.

Case 1 in Proposition 1.7 refers to the situation where total profits decrease when the constraint is introduced, $T^{\mathrm{NC}} \cdot \hat{c}_{b,0}^{\mathrm{NC}} \geq T_b^{\mathrm{C}} \cdot \tilde{c}_{b,0}$. This situation can arise in two scenarios. First, when the intensive and extensive margins both decrease total profits (i.e. $\Delta_K \leq 0$). Second, when the intensive margin increases profits (i.e. $\Delta_K > 0$) but the extensive margin dominates. As profits decrease, the wealth effect in the saver's allocation implies a decrease in her non-durable consumption $\left(\tilde{c}_0^s \leq \hat{c}_{s,0}^{\mathrm{NC}}\right)$ and an increase in her labor supplied $\left(\tilde{n}_0^s \geq \hat{n}_{s,0}^{\mathrm{NC}}\right)$. Hence, at the prices $\widehat{\mathbf{P}}^{\mathbf{NC}}$, both agents want to consume less and are willing to work more; resulting in excess supply in the initial period's labor market.

Case 2 in Proposition 1.7 refers to the situation where total profits increase, $T^{\mathrm{NC}} \cdot \hat{c}_{b,0}^{\mathrm{NC}} < T_b^{\mathrm{C}} \cdot \tilde{c}_{b,0}$.

[14]For the savers, the partial equilibrium allocation does not change, hence $T_s^{\mathrm{NC}} = T_s^{\mathrm{C}}$.

This situation happens only if the intensive margin increases profits (i.e. $\Delta_K > 0$) and it dominates the extensive margin. Larger profits imply a larger lifetime wealth for the saver; which results in larger consumption $\left(\tilde{c}_0^s \geq \hat{c}_{s,0}^{\text{NC}}\right)$ and smaller labor supplied $\left(\tilde{n}_0^s \leq \hat{n}_{s,0}^{\text{NC}}\right)$. At the prices $\widehat{\mathbf{P}}^{\mathbf{NC}}$, the two types of agents have now opposite reactions to the introduction of the credit constraint. The aggregate result is an excess supply of labor as long as the savers have a large enough savings motive (κ_s) and initial wealth $\left(\overline{W}_{-1}^s\right)$. For large enough κ_s, most of the increase in the saver's lifetime wealth translates into an adjustment in future consumption and labor supplied rather than in the current ones. The condition that the savers' initial wealth is large enough, $\overline{W}_{-1}^s \geq \Omega_s^{-1}\hat{c}_{b,0}^{\text{NC}}\left[\Omega_s K_C^s + \Omega_b\left(K_C^b - K_R^b\right)\right]$, ensures that changes in profits account only for a small portion of lifetime wealth, hence leading to small adjustments in consumption and labor supplied. Given these two conditions, the increase in savers' current consumption is moderate relative to the borrowers' decrease; the result is a decrease in aggregate consumption and hence in labor demanded. Similarly, the decrease in savers' current labor supplied is moderate relative to the borrowers' increase; the result is an increase in aggregate labor supplied. Thus these two conditions are enough to guarantee an excess of labor supplied in the initial period at the prices $\widehat{\mathbf{P}}^{\mathbf{NC}}$.

This concludes the partial equilibrium analysis. The immediate question is, how do prices (specifically the wage rate) adjust in order to reestablish equilibrium in the labor market? The following section provides a numerical exercise in order to illustrate this general equilibrium effects.

1.4.2 General Equilibrium: A Numerical Exercise

The following section illustrates the general equilibrium effects of introducing the credit constraint. I first show that the theoretical Propositions 1.3 - 1.7 made in Section 1.4.1 hold in this numerical exercise. I then proceed to illustrate the price adjustments that take place in order to bring markets back to equilibrium. Finally, I end the section by showing the effect that the price adjustments have on the agents' equilibrium allocations and in aggregate output.

Calibration

The numerical exercise is conducted for a range of monetary policy shocks $\epsilon_0^z \in [-1, 1]$. This shock is measured in percentage points of the nominal interest rate. Hence, $\epsilon_0^z \in [-1, 1]$ refers to a ϵ_0^z percentage points surprise decrease/increase of the nominal interest rate.

Table 1.1: Table of Parameter Values and Initial State

Calibrated	
Parameter	**Value**
β_s	0.99
δ	0.01
ν_s	4.92
ν_b	8.23
ϵ_j	5.00
χ	0.29
α	0.69
ϕ_c	0.75
ϕ_d	0.50

Arbitrary	
Parameter	**Value**
β_b	0.98
θ	1.00
ϕ_π	1.50
ρ	0.50
γ	0.50
κ_b	12.45
κ_s	76.75

Initial State	
Variable	**Value**
$R_{-1}b^b_{-1}$	3.10
$R_{-1}b^s_{-1}$	-3.10
d^b_{-1}	4.40
d^s_{-1}	8.88
q_{-1}	1.00
$\Delta_{c,-1}$	1.00
$\Delta_{d,-1}$	1.00

The model's parameters are calibrated by imposing that several key moments match their empirical counterparts.[15] Table 1.1 summarizes the corresponding parameter values and the initial state of the economy. The empirical targets are based on quarterly data for the U.S. economy. The annual interest rate is targeted at 3%, which implies that $\beta_s = 0.99$. The annual depreciation rate of the durable good is set at 4%, which pins down the value of $\delta = 0.01$. Both types of agents are assumed to supply one third of their total time endowment (normalized to one); which implies labor disutility parameters of $\nu_s = 4.92$ and $\nu_b = 8.23$. The other preference parameter, $\alpha = 0.69$, is calibrated so that the total share of private spending on the durable good is 20%. The parameter $\chi = 0.29$, which controls the fraction of the durable good that can be pledged as collateral, is determined by requiring a loan-to-value ratio of 70% for the borrowers.

On the production side, ϕ_j is chosen to match the average price adjustment frequencies in the durable and non-durable sectors. I assume that the durable sector can adjust prices more frequently, with an average adjustment frequency of two quarters. The non-durable sector adjusts its prices every four quarters.[16] Therefore, $\phi_c = 0.75$ and $\phi_d = 0.5$. In addition, and to simplify the analysis, both sectors are assumed to have the same price markup, which is set at 20%, implying that $\epsilon_c = \epsilon_d = 5$.

The rest of the parameters are fixed at arbitrarily chosen values. I only consider the symmetric case where the mass of borrowers and savers in the economy is equal $\Omega_s = \Omega_b = 0.5$. Following

[15]The moments used for the calibration correspond to the deterministic steady state of the infinite horizon version of the model. Refer to Chapter 2 for the complete specification of this infinite horizon version of the model.

[16]This is somewhat arbitrary. There is no clear consensus in the price stickiness literature as to which sector is relatively more sticky.

Krusell and Smith (1998), the preference discount factor for the borrowers is set to $\beta_b = 0.98$. It is also assumed that the monetary authority weights the durable and non-durable sectors equally when computing the composite inflation index, so that $\gamma = 0.5$. To satisfy the Taylor principle, I set $\phi_\pi = 1.5$. Finally, the persistence of the monetary policy shock is given by $\rho = 0.5$. This is perhaps the only parameter value that is not within the range of standard values in the literature. In order to match some second moments of the data, the persistence of the monetary policy shock is usually much larger, $\rho \sim 0.95$. However, the value of this parameter does not significantly impact the goal of the chapter; highlighting the mechanisms by which credit constraints alter the equilibrium of this economy.

Finally, the initial wealth distribution and prices are held fixed at the steady state levels for the infinite horizon version of the model. Thus the parameters $\kappa_b = 12.45$ and $\kappa_s = 76.75$ are set in order to ensure that the equilibrium of the two period baseline economy is as close as possible this steady state.

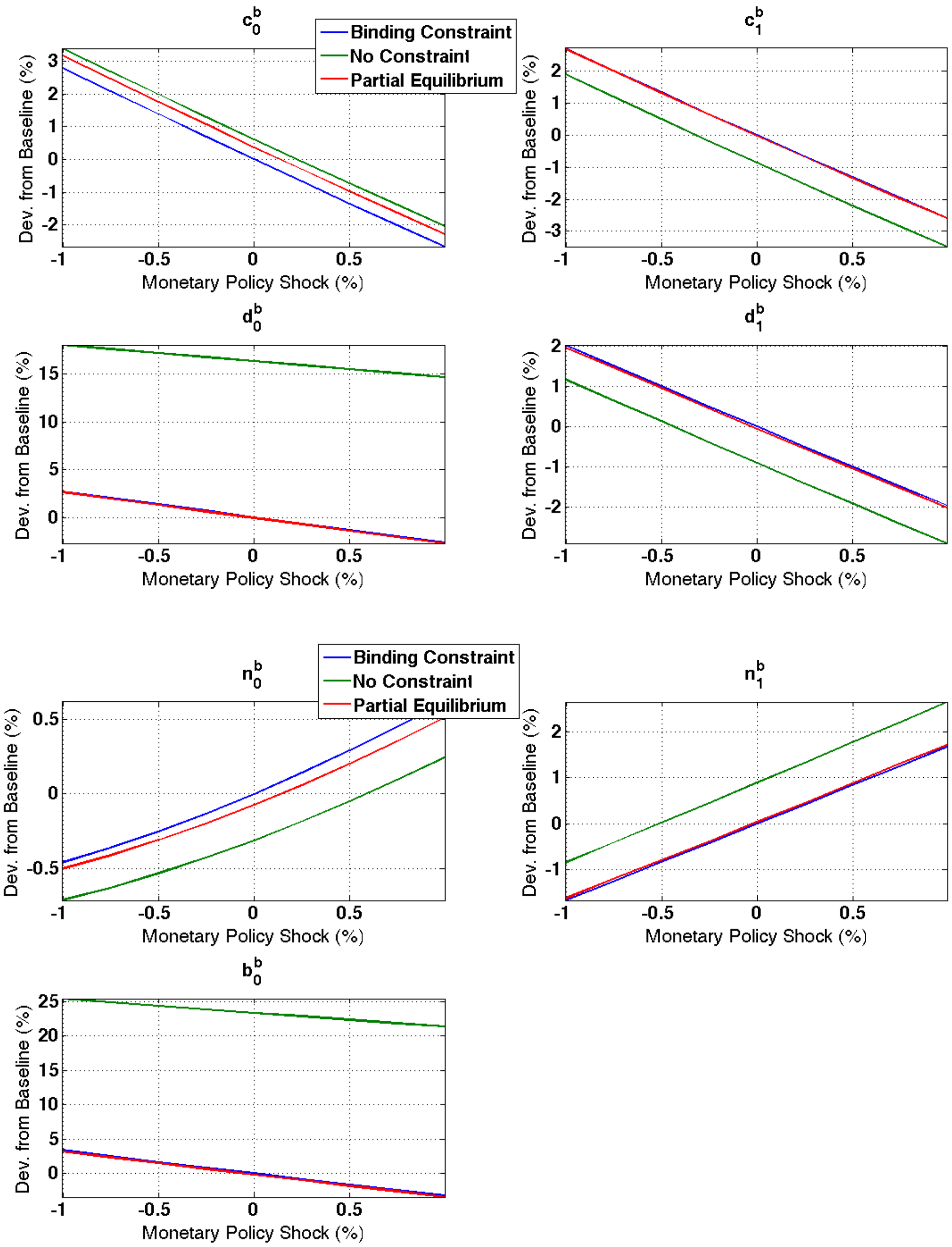
Partial Equilibirum

In what follows, I define the baseline economy as the economy with credit constraint ($E = C$) and with no monetary policy shock ($\epsilon_0^z = 0$). All the comparison and figures are done relative to this baseline case.

Figures 1.1 - 1.2 illustrate the percentage deviation from the baseline economy for the borrower's and saver's allocations as a function of the monetary policy shock. Each figure has three lines corresponding to the equilibrium of the economy with credit constraints (the blue line labeled "Credit Constraint"), the equilibrium of the economy without credit constraints (the green line labeled "No Constraint"), and the partial equilibrium of the economy with credit constraints given the equilibrium prices of the economy without credit constraints (the red line labeled "Partial Equilibrium"). Thus the figures decompose the effect of the credit constraints into the partial equilibrium effect (going from the "No Constraint" to the "Partial Equilibrium") and the general equilibrium effect (going from the "Partial Equilibrium" to the "Credit Constraint").

The partial equilibrium effect of the credit constraint for the borrower's allocation is summarized in Figure 1.1. Given the current parameterization, the borrower's initial wealth $\left(W_{-1}^b\right)$ is sufficiently small and the conditions of Proposition 1.3 hold. For any level of the monetary policy shock, the

Figure 1.1: Borrower's Allocation



credit constraint is binding. The extent to which the constraint is binding is illustrated in the panel labeled "b_0^b". Roughly speaking, the difference between the debt holdings in an economy with and without credit constraints is about 20% of the baseline level of debt.

As stated in Proposition 1.4, the binding constraint introduces two effects. First, a wealth effect due the borrower's reduced ability to transfer wealth. Second, a substitution effect given the larger effective price of the durable good. The combination of these two effects determines the partial equilibrium change in the borrower's allocation. For the initial period's durable good, both effects complement each other leading to a large decrease in consumption, as show in the panel "d_0^b". For the remaining variables, the wealth and substitution effects oppose each other; however, the wealth effect dominates. In terms of consumption, this implies a decrease in the initial period's non-durable consumption (panel "c_0^b") and an increase in consumption of both types of goods in the final period (panels "c_1^b" and "d_1^b"). In terms of the labor supplied, the wealth effect leads to a decrease in the final period's labor supplied and an increase in current labor supplied, as evinced in panels "n_1^b" and "n_0^b".

From Proposition 1.7, the change in the borrower's partial equilibrium allocation affects the intermediate firms' profits via two margins. The intensive margin, which in the current case leads to an increase in the profits per unit of non-durable good sold to the borrower; and the extensive margin, which tends to decrease the firms profits as the borrowers decrease their consumption of non-durable good. For the current parameterization, the dominant effect is that of the intensive margin; the saver's non-labor lifetime wealth increases due to the increase in intermediate firms' profits.

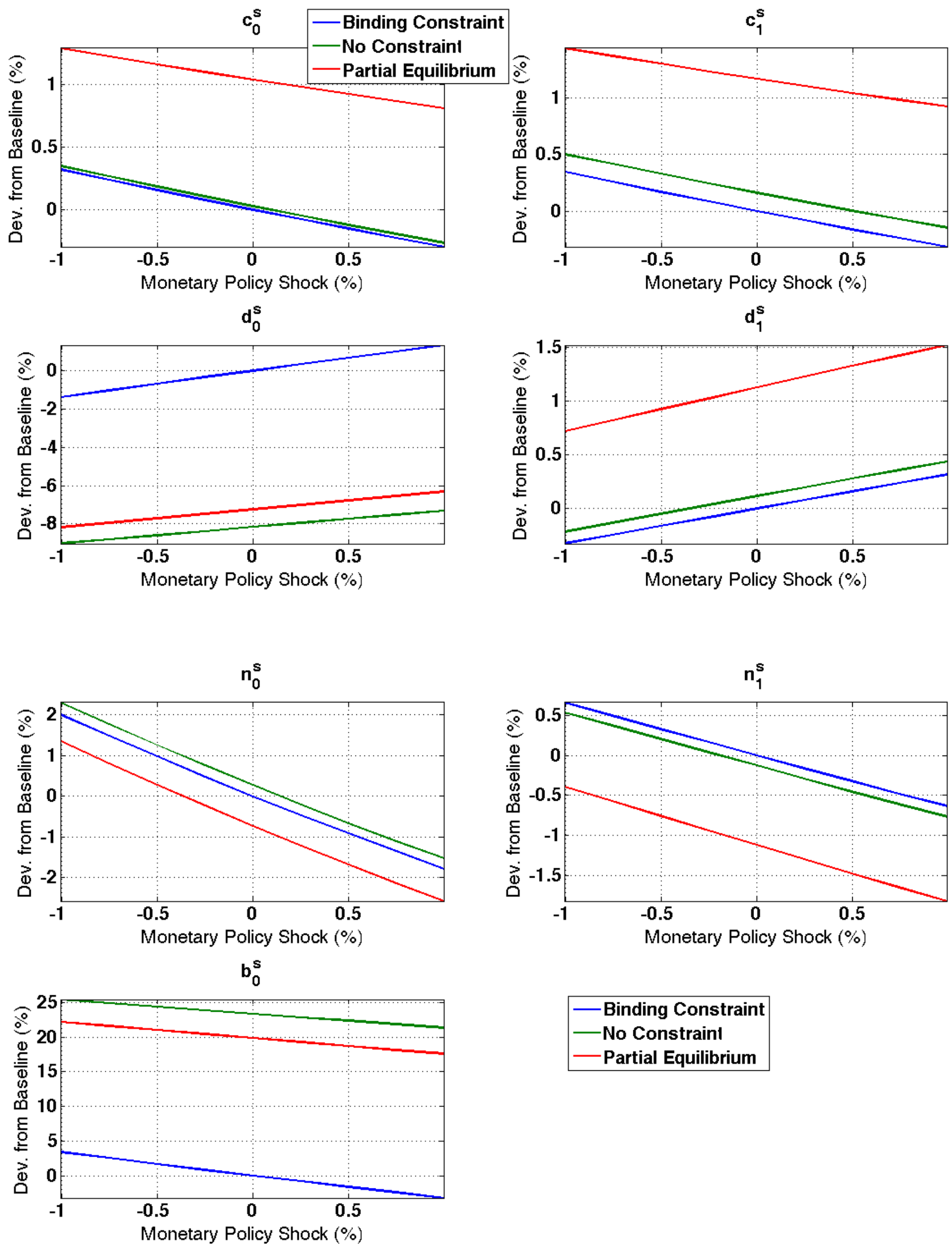
Figure 1.2 illustrates the effect of this increase in lifetime wealth on the saver's allocation. The consumption of the durable and non-durable goods increases in both periods, as shown in panels "c_0^s", "d_0^s", "c_1^s" and "d_1^s". This increase in consumption is rather homogeneous across all variables and amounts to about 1% of their respective baseline levels. As for the labor supplied, panels "n_0^s" and "n_1^s" show that the saver works less in both periods when the credit constraint is introduced.

General Equilibrium: Effect on Prices

The general equilibrium price adjustments can be understood by analyzing the labor market in the initial period. The model's parameterization guarantees that the conditions of Proposition 1.7 hold. That is, the saver's initial wealth $\left(\overline{W}^s_{-1}\right)$ and durable continuation value (κ_s) are large enough to ensure an excess supply in the initial period's labor market.

Consider first the total aggregate labor demanded by the firms. For the non-durable sector, the borrowers' decrease in current consumption and the savers' increase in current consumption are roughly of about 1% of their respective baseline levels. Since the savers' baseline consumption level

Figure 1.2: Saver's Allocation



is larger, this implies a modest increase in aggregate non-durable demand. In turn, this increase in production must be met by a modest increase in the labor demanded by this sector. For the durable sector, borrowers decrease durable consumption by about 15%. Savers, on the other hand, increase their durable consumption by a modest 1%; the durable sector experiences a substantial decrease in aggregate demand. This decrease in production implies a large decrease in the labor demanded by this sector. Overall, the total labor demanded by the firms in the initial period decreases.

Similarly, the change in total aggregate labor supplied depends on the relative size of the changes in labor supplied by the borrowers and savers. Labor supplied increases by about 0.25% for the borrowers and decreases by about 1% for the savers, both with respect to their baseline levels. However, given the current parametrization, the baseline level of labor supplied by borrowers is larger than that supplied by the savers. In the end, these adjustments lead to an increase in total aggregate labor supplied.

Given the equilibrium wage rate of the economy without credit constraints $\left(\hat{\omega}_{c,0}^{\mathrm{NC}}\right)$, the decrease in aggregate labor demanded along with the increase in aggregate labor supplied imply an excess supply of labor. In a nutshell (and as pointed out in Proposition 1.7), for large enough saver's durable continuation value and initial wealth, the changes in the initial period's optimal allocation of the savers are modest compared to the changes in the borrowers' allocation. Thus the borrowers' decrease in consumption and increase in labor supplied are responsible for the excess supply in the labor market.

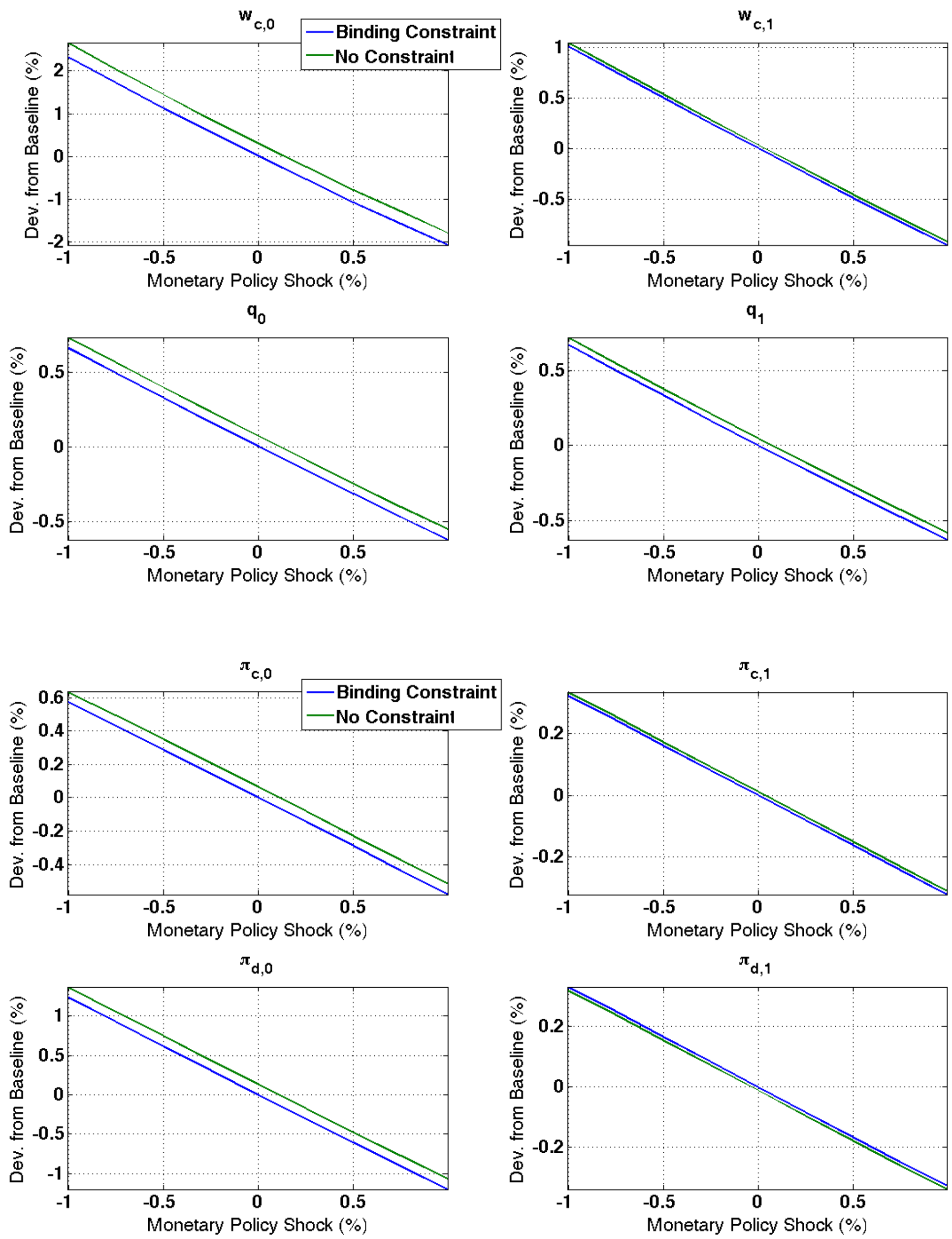
The resulting general equilibrium price adjustments are summarized in Figure 1.3. Consider first the initial period real wage rate; it decreases in order to clear the labor market. This situation can be clearly seen in the top-left panel labeled "$w_{c,0}$". Given the model's assumptions, the aggregate labor demand is downward slopping and the aggregate labor supply is upward slopping.[17] Therefore, the excess demand $N_0^{\mathrm{C}}\left(\hat{\omega}_{c,0}\right)$ is a decreasing function of the real wage rate. That is, the real wage rate must decrease in order to eliminate the labor market's excess supply generated by the introduction of the credit constraint. Although I don't provide any formal proof for this result, I summarize it in the following conjecture.

Conjecture 1.8 *Fix z_0, $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$, and $\left(R_{-1}b_{-1}^a, d_{-1}^a\right)_{a\in\{s,b\}}$. The equilibrium real wage rate in units of the non-durable good decreases when introducing the credit constraint: $\hat{\omega}_{c,0}^{C} \leq \hat{\omega}_{c,0}^{NC}$.*

Consider next the remaining initial period prices (which are all shown in the left panel of Figure 1.3). The lower real wage rate implies smaller real costs for the initial period in both, durable and non-durable production. Since intermediate firms optimally set their price as a fixed mark-up over the present discounted marginal costs, they now set lower prices. Therefore, sectorial inflation

[17] At least in a neighborhood of the equilibrium wage rate of the economy without credit constraints, $\hat{\omega}_{c,0}^{NC}$.

Figure 1.3: Price Adjustments



decreases in the initial period. The panels labeled "$\pi_{c,0}$" and "$\pi_{d,0}$" illustrate this decrease. It is obvious from the figure that the decrease in inflation is larger in the durable sector. This is a consequence of the assumption that this sector is more flexible than the non-durable one ($\phi_c > \phi_d$). Finally, due to the larger decrease in durable inflation, the relative price q_0 decreases.

The general equilibrium effect on final period prices is illustrated in the right panels of Figure 1.3. The intuition for the adjustments is very similar to the initial period prices; a decrease in the real wage rate (panel "$w_{c,1}$") leads to a decrease in prices in both sectors. Given the assumption that durable sector is more flexible, this implies a decrease in the relative price of durables (panel "q_1").

From the previous discussion it is clear that the sectorial price *levels* decrease in both periods. Therefore, the change in sectorial inflation depends on the relative size of these decreases between periods. This relative size is determined by the assumptions that the monetary authority fixes the interest rate R_1 in the second period (which effectively implies perfect inflation targeting by fixing the composite inflation index), and the assumption that the durable sector is more flexible. For the non-durable sector, these assumptions imply that the second period price level decrease dominates; thus non-durable inflation decreases when the credit constraint is introduced (panel "$\pi_{c,1}$"). For the durable sector, the dominant price level decrease occurs in the first period; hence durable inflation increases as shown in panel "$\pi_{d,1}$". Finally, note that all the price adjustments are substantially smaller in the second period, which is a consequence of the low persistence of the monetary shock.

General Equilibrium: Price Adjustment Effect on Agents' Allocations

The adjustment of the borrowers' and savers' allocations to the general equilibrium price changes is illustrated in Figures 1.1 and 1.2. In particular, the effect can be visualized as the difference between the "Partial Equilibrium" (red) and "Binding Constraint" (blue) lines.

Consider first the borrowers' initial period allocation, which is a function of the real value of the inherited wealth, the initial period's real wage rate, and the effective durable price.[18] Thus the general equilibrium price adjustments lead to income and substitution effects. The resulting income effect is negative, driven by a decrease in both labor earnings and inherited wealth. On one hand, the decrease in labor earnings is a consequence of the decrease in the initial period's real wage rate. On the other hand, the decrease in inherited wealth is due to two factors. First, the smaller non-durable inflation in the initial period leads to an increase in the real value of the inherited debt obligations. Second, the smaller relative price of the durable good implies a lower market value of the inherited durable holdings. As for the substitution effects, they operate along two dimensions; substitution

[18]Recall that the effective durable price is defined as $q^{\mathrm{C}}_{0,\mathrm{eff}} \equiv q_0 \left(1 - (1-\chi)(1-\delta)\,\pi_{d,1} R_0^{-1}\right)$ for the borrowers and $q^{\mathrm{NC}}_{0,\mathrm{eff}} \equiv q_0 \left(1 - (1-\delta)\,\pi_{d,1} R_0^{-1}\right)$ for the savers.

between leisure/consumption and between durable/non-durable consumption. The decrease in the real wage rate makes leisure more attractive relative to consumption, thus providing an incentive for borrowers to decrease their labor supplied. Similarly, the decrease in the effective durable price (which is driven by the decrease in the relative price q_0 and the increase in durable inflation $\pi_{d,1}$ in the final period) makes durable consumption relatively more attractive than non-durable one.

Overall, the effect on the borrowers' initial period allocation is given by the relative size of the income and substitution effects. Non-durable consumption decreases, labor supplied increases, and durable consumption is unaffected, as illustrated in panels "c_0^b", "n_0^b", and "d_0^b" of Figure 1.1. The negative income effect, the substitution of leisure for consumption, and the substitution of durable for non-durable consumption ensure that non-durable consumption decreases. The increase in the labor supplied is a consequence of the negative income effect dominating the substitution of leisure for consumption. Finally, there is no general equilibrium change in durable consumption given that, under the current parameterization, the substitution and income effects end up cancelling one another.[19]

Additionally, as it can be seen from panel "b_0^b" in Figure 1.1, the general equilibrium change in borrowers' debt holdings is negligible. In the economy with credit constraints, the borrowers debt holdings are determined by the repayment interest rate and the market value of the collateral. The later depends on two factors, the amount of durable holdings and the durable's market price. From the previous discussion, it is clear that the general equilibrium adjustment of durable holdings is negligible. Hence, debt holdings are determined entirely by the relationship between the change in the nominal interest rate and the market price of the durable good. The nominal interest rate decreases, a consequence of the market's response to provide savers with enough incentive to reduce their nominal bond holdings. Similarly, the real price of the durable good decreases, which is mostly driven by the decrease in the durable's relative price. Thus the net effect is that the amount of debt holdings does not significantly change and it is pretty much unaffected by the general equilibrium price adjustments.

Consider next the borrowers' final period allocation. As it can be seen from the panels "c_1^b", "n_1^b", and "d_1^b" in Figure 1.1, the change in this allocation is negligible. The price adjustments

[19] If anything, the substitution effects slightly dominates, resulting in a very modest (almost negligible) increase in durable consumption.

are small and lead to modest income and substitution effects, which end up cancelling each other anyways. Additionally, the wealth effects that could potentially arise from wealth transfer from the initial to the final period are negligible as the adjustment in the borrowers asset position (durable and debt holdings) is very modest, as discussed in the previous paragraphs.

I move now to the effect of the general equilibrium price adjustments on the saver's allocation, which are shown on Figure 1.2. Given savers face no credit constraints, their consumption and labor supplied (for both periods) are functions of the agent's lifetime wealth and the goods' prices. Therefore, the general equilibrium price adjustments lead, again, to income and substitution effects.

In each of the two periods, the intra-temporal substitution effects operate along two dimensions; substitution between leisure/consumption and between durable/non-durable consumption. The decrease in the each period's real wage rate ($w_{c,0}$ and $w_{c,1}$) makes leisure more attractive relative to consumption. Similarly, the decrease in the each period's effective durable price ($q_{0,\text{eff}}^{\text{NC}}$ and q_1) makes durable consumption relatively more attractive than non-durable one.

In addition, one can also think of an inter-temporal substitution effect. On one hand, current consumption become more attractive for the savers as the nominal interest rate decreases; a consequence of debt market clearing. On the other hand, future consumption becomes more attractive as non-durable inflation in the final period decreases. In the end, the interest rate adjustment dominates and the result is that savers have an incentive to substitute current consumption for future consumption.

The income effect for the savers is more complex than for the borrowers as their lifetime wealth depends on three factors; lifetime labor earnings, effective initial wealth, and profits from the intermediate firms. The overall result is a negative income effect, which can be better understood by analyzing the impact of the general equilibrium price adjustments on each of these three factors. First, the general equilibrium price adjustments lead to a decrease in lifetime labor earnings; a consequence of the decrease in the real wage rate for each period. Second, the real value of the inherited wealth decreases as well. Despite lower inflation increasing the saver's real value of the nominal bond holdings, the large decrease in the real value of the inherited durable holdings (driven by the decrease in q_0) is responsible for the smaller real value of the inherited assets. Lastly, the general equilibrium price adjustments end up increasing the intermediate firms' profits. Although the general equilibrium adjustment in the borrowers allocation implies firms end up selling less of the goods, the increase in profits is driven by an increase in the profits per unit sold by each sector.[20] The larger profits per unit sold are a consequence of the smaller real wage rate firms have to pay in each period. In the end, the decrease in lifetime labor earnings and inherited wealth overpower the increase in profits, leading to the negative income effect.

With the understanding of all these general equilibrium effects, consider now the savers' initial period allocation. On one hand, panel "c_0^s" of Figure 1.2 shows how non-durable consumption decreases driven by the negative income effect and the intra-temporal substitution of leisure for consumption and of durable consumption for non-durable consumption. On the other hand, durable consumption increases as shown on panel "d_0^s" of Figure 1.2; the substitution effects, both intra-temporal durable good for non-durable good and inter-temporal current consumption for future consumption, are larger than the negative income effect. Lastly, labor supplied increases as evinced on panel "n_0^s" of Figure 1.2. Again, this response to the general equilibrium price adjustments is mostly a consequence of the negative income effect, which dominates the intra- and inter-temporal substitution effects.

Finally, consider the savers' final period allocation. The combination of the negative income effect and the inter-temporal substitution effect implies savers react to the general equilibrium price changes by decreasing their final period's consumption; as shown on panels "c_1^s" and "d_1^s" of Figure 1.2. Additionally, the small intra-temporal consumption/leisure effect ensures the increase in leisure is modest, thus ensuring labor supplied increases; as seen on panel "n_1^s".

General Equilibrium: Overall Effect on Aggregate Output

The effect of the credit constraint on aggregate output can be understood by adding together the partial and general equilibrium effects on the borrowers' and savers allocations'. The total effect of the credit constraint on the agents' allocations is illustrated in Figures 1.1 and 1.2 as the difference between the "No Constraint" (green) and the "Binding Constraint" (blue) lines. Overall, the adjustment in the borrowers' and savers' allocations is dominated by wealth effects.

For borrowers, the constraint effectively decreases their wealth in the initial period and increases it in the final period. This is because the binding constraint limits the amount they can borrow against

[20] As previously discussed, and as evinced in Figure 1.1, the general equilibrium price adjustments imply borrowers decrease their non-durable consumption in the initial period without significantly adjusting their remaining consumption variables.

their future income. Therefore, consumption in the initial period decreases while it increases in the final period; this is true for the durable good, non-durable good, and leisure.

For savers, lifetime wealth decreases. For the final period variables, this wealth effect is dominant; which explains the decrease in consumption and the increase in labor supplied. For the initial period variables, the wealth effect is dominant only for non-durable consumption. For durable consumption and labor supplied, the substitution effect is larger as it is a combination of an intra- and inter-temporal effects. The intra-temporal substitution is a consequence of the decrease in the real wage rate and the durable's relative price; savers favor consumption of leisure and durable good over the non-durable. The inter-temporal substitution effect arises as a consequence of the decrease in the nominal interest rate R_0; savers favor current consumption over future one. That is, savers find current consumption more attractive, especially leisure and durable consumption. Note that the nominal interest rate R_0 decreases in order to ensure the nominal bond market clearing; as borrowers can access less debt, savers must have sufficient incentive to stop using nominal debt as a savings channel.

The adjustment in the borrowers' and savers' allocations leads to an adjustment in aggregate sectorial output, as summarized by the following conjecture:

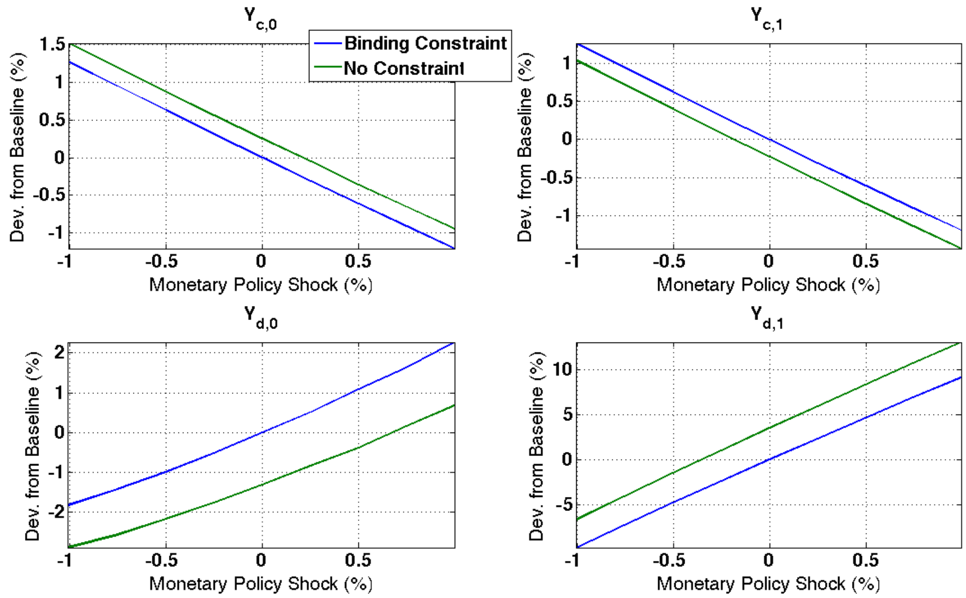
Conjecture 1.9 *Fix z_0, $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$, and $\left(R_{-1}b^a_{-1}, d^a_{-1}\right)_{a\in\{s,b\}}$. Given that $\hat{\omega}^C_{c,0} \leq \hat{\omega}^{NC}_{c,0}$, then:*

(a) $Y^{NC}_{c,0} \geq Y^C_{c,0}$ *and* $Y^{NC}_{d,0} \leq Y^C_{d,0}$

(b) $Y^{NC}_{c,1} \leq Y^C_{c,1}$ *and* $Y^{NC}_{d,1} \geq Y^C_{d,1}$

These results are illustrated in Figure 1.4. As a consequence of the credit constraint, the initial period's output decreases in the non-durable sector while it increases in the durable one (left panels of Figure 1.4). The former is a consequence of the decrease in non-durable consumption by both, borrowers and savers. The later is driven by the increase in the savers' durable consumption. Intuitively, since the credit constraint limits the amount of nominal debt for the borrowers, the savers must find an alternative savings channel; the durable good. The incentive for the savers to use the durable good is achieved mostly via the general equilibrium price adjustments; the large decrease in the durables relative price q_0 and the decrease in the nominal interest rate R_0.

Figure 1.4: Equilibrium sectorial output as a percent of baseline case



For the final period's sectorial output, the situation is somewhat reversed; non-durable output increases while durable output decreases (right panels of Figure 1.4). In this period, savers' and borrowers' have opposite reactions; savers decrease consumption of both goods while borrowers increase it. Note however that given the more impatient nature of borrowers, non-durable consumption is relatively more important in their consumption bundle. Similarly, since savers are more patient, durable consumption is relative more important in their consumption bundle. Hence, non-durable output increases driven by the borrowers' consumption adjustment and durable output decreases driven by the savers consumption adjustment.

1.5 Conclusion

Using a simple two-period, two-agent version of the baseline two-sector New Keynesian model, I investigate the effect of credit constraints on aggregate sectorial output. I introduce the credit constraint as a collateral borrowing constraint faced only by one type of agents, the borrowers. The other type of agents, the savers, face no credit constraint and have some ownership of the firms in the economy.

The aggregate effects of the credit constraint are non-trivial; they are the result of the interaction between the agents' response to the credit constraint. The adjustment of the agents' allocation is in turn determined by competing wealth and substitution effects. Overall, the constraint affects agents via two channels. First, the direct channel; borrowers must adjust their allocation once the constraint is introduced as the no-constraint allocation might be unfeasible. In turn, this affects the savers' allocation; there is a wealth effect due to the change in firms' profits associated with the change in borrowers' allocation. Second, the general equilibrium channel; prices adjust in order for markets to clear and agents react to these prices changes.

For the borrowers, the general equilibrium price changes are relatively unimportant; they adjust their allocation mostly to ensure it is feasible given the constraint. However, for the savers, the general equilibrium price changes lead to a significant adjustment in their allocation. For instance, the adjustment in the durable's relative price and the nominal interest rate are responsible for the savers' increase in the initial period's durable consumption, which is the main driver of the increase in the initial period's aggregate durable output.

The main takeaway of the chapter is really simple; the credit constraint (and financial frictions for that matter) have important indirect general equilibrium effects. Despite the constraint affecting only borrowers directly, it does have an important indirect impact on the savers' allocation. This indirect effect is mainly a consequence of general equilibrium price adjustments. Therefore, models which use a representative agent to study the effects of financial frictions might not capture these general equilibrium effects that arise due to adjustments in prices and profits. This general equilibrium effects are not only important to the extent that they might affect particular agents in the economy, but also given that they can have an impact on aggregate outcomes.

CHAPTER 2

Monetary Policy and Credit Constraints

Abstract

I investigate the effect of credit constraints on the monetary policy transmission mechanism using a standard two-sector New Keynesian model. The credit constraint is introduced as a collateral constraint that limits borrowing via a nominal bond. There are two types of agents; only the relatively more impatient agents face the collateral constraint. I find that the inclusion of credit constraints can potentially lead to very different conclusions about the effects of a monetary expansion relative to the canonical New Keynesian model with no financial frictions; when the mass of credit constrained agents is sufficiently large, a monetary "expansion" can actually lead to a contraction of total output. In the model with credit constraints, the transmission mechanism is mostly a consequence of the general equilibrium price adjustments rather than a consequence of the direct inter-temporal substitution effect. Overall, my findings highlight the importance of the indirect effects of monetary policy transmission which operate via the market value of the collateral, firms' profits, and asset portfolio adjustments.

JEL Codes: E21, E44, E52

2.1 Introduction

This chapter intends to elaborate and expand on the insights and results obtained in the first chapter. Recognizing the limitations of the simple two-period model, the current chapter relaxes this assumption and considers the infinite horizon version instead. As before, the main goal is to shed some light on the monetary policy transmission mechanism when augmenting the standard New Keynesian setting to incorporate financial frictions.

A vast literature has extensively studied the macroeconomic implications of standard versions of the New Keynesian model. Authors such as Clarida, Galí and Gertler (2000) and Woodford (2001), focus on the study of simple versions of the model that have price stickiness as the only nominal friction. Some other authors, like Christiano and Evans (2005), consider slightly augmented versions that incorporate additional nominal frictions. Furthermore, extensive reviews that explain in detail the theorical foundations of the New Keynesian framework and its applications to monetary policy analysis are readily available, for instance Woodford (2003), and more recently, Galí (2008).

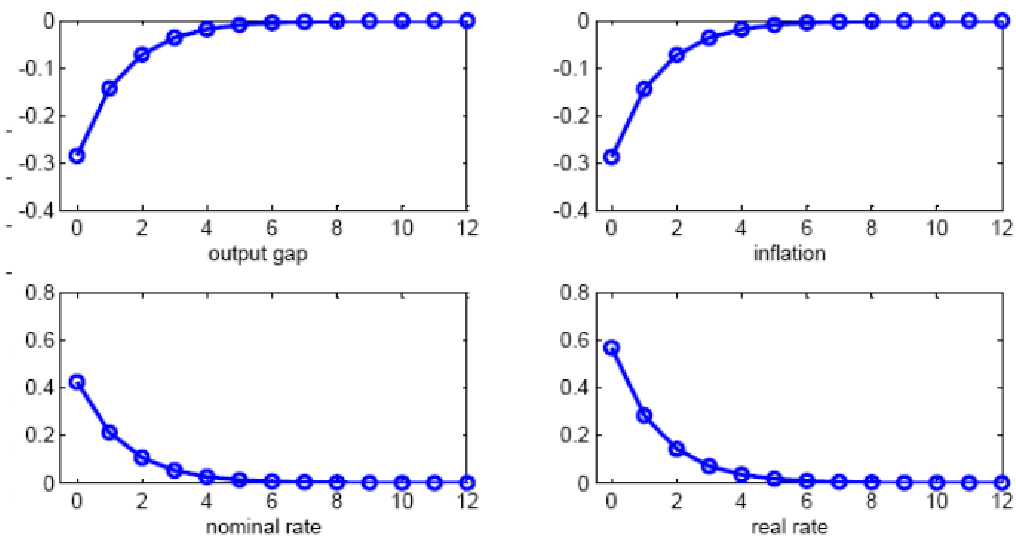
One of the main findings of such literature relates to the way in which monetary policy affects total aggregate output in the economy. In particular, under the assumption that monetary policy is conducted through a Taylor-type rule, the consensus is that surprise shocks to this rule are negatively correlated with output. That is, a positive surprise shock to the monetary policy rule results in an increase in the real interest rate and a decrease in aggregate output; as shown in Figure 2.1.[21] The mechanism behind such response is driven by the inter-temporal substitution effect implied by the change in the real interest rate.

To some extent, this result is the underlying corner stone of the believe, shared by many academics, that the monetary authority can help stimulate or contract the economy through this interest rate channel. Indeed, the term "conventional monetary policy" has been coined to refer to policy interventions that try to exploit this mechanism. However, the 2008 financial crisis in the U.S. forced an entirely new perspective on this view. The so called conventional monetary policy measures taken by the Federal Reserve did not work as expected. Since then, many authors have tried to evaluate the effectiveness of conventional monetary policy interventions.[22] This literature emphasizes the increasing importance of the financial sector in the transmission of monetary policy to the

[21] A negative surprise shock would lead to a decrease in the nominal rate and an increase in output.

[22] See for instance del Negro et al. (2011) and Mishkin (2009).

Figure 2.1: Effect of a Monetary Policy Shock (Interest Rate Rule)



Note: Figure taken from Galí (2008).

macroeconomy, which the basic New Keynesian model completely ignores.

Motivated by these observations, the present chapter revisits the standard New Keynesian model incorporating to it one of the most basic financial frictions; a collateral constraint. I argue that even in this most simple setting, there is an important channel which has been largely ignored by the literature; the indirect general equilibrium effects. In the presence of the constraint, the general equilibrium price adjustments affect the market value of the good that is pledged as collateral; hence the degree to which the constraint is binding. Borrowers are then forced to adjust their allocation; this causes a change in firms' profits which is responsible for the wealth effect that leads to an adjustment in the savers' allocation. Additionally, the general equilibrium price changes induce a portfolio reallocation for the savers. The combination of all of these effects dominates the usual inter-temporal substitution effect that drives the aggregate output response in the standard New Keynesian model.

Figure 2.2 illustrates the effect on total output of a negative surprise shock of 25 b.p. to the monetary policy rule for the two limiting cases of the model studied in this chapter; when none of the agents face the collateral constraint (the case $\Omega_b \to 0$), and when all of the agents face it (the case $\Omega_b \to 1$). For the case $\Omega_b \to 0$, the model reduces to that presented in Galí (2008), yielding the standard result that a negative shock to the nominal interest rate rule translates into an economic stimulus. However, in the limiting case where all agents are credit constrained, the effect of a conventional monetary policy intervention is significantly different from that predicted by the standard model; the negative shock to the nominal interest leads to a contraction of aggregate output for all periods after impact.

The rest of the chapter is organized as follows. Section 2.2 specifies in detail each of the components of the model. Section 2.3 presents a characterization of the models' solution. Section 2.4 presents the models' calibration and the main results of the paper. Finally, the concluding remarks are presented in Section 2.5.

2.2 Model

The model is the infinite horizon extension of the one presented in Chapter 1. The economy is populated by two types of agents who differ along three key dimensions: their degree of patience, the ownership of firms, and their ability to access the financial market. Agents work, consume, and accumulate wealth using two instruments; a durable good and a nominal risk free bond. The agents with the smallest time preference discount parameter (*borrowers*), face a collateral credit constraint on their nominal debt holdings.[23] The agents with the largest time preference discount parameter (*savers*) can freely access the nominal bond market as long as a No-Ponzi condition is satisfied.[24] The savers are the owners of the firms that produce the intermediate varieties. The intermediate firms use labor as the only input in production. There are two final consumption goods, durables and non-durables, which are produced using the intermediate varieties. Finally, monetary policy is conducted through a nominal interest rate rule of the type first introduced by Taylor (1993). The only non-trivial source of uncertainty is an exogenous stochastic process in the nominal interest rate rule; which captures unanticipated actions by the monetary authority.

[23]This constraint is of the type proposed by Kiyotaki and Moore (1997).

[24]Despite the use of the terms borrowers and savers, both types of agents are free to borrow or save using the nominal bond. However, in the deterministic steady state solution the more patient agents lend resources to the impatient agents; effectively the more patient agents are savers and the less patient ones are borrowers.

2.2.1 Producers of Final Goods

There are two production sectors denoted by $j \in \{c, d\}$; where c refers to the non-durable sector and d to the durable one. Firms in sector j operate under a perfectly competitive environment. Each firm in sector j uses as inputs a continuum of differentiated goods, which it must buy from the intermediate firms. Let $i \in [0, 1]$ be the index of each intermediate firm. Hence $y_{j,t}(i)$ is the variety produced by the i^{th} intermediate firm in sector j to be used in the production of final good $Y_{j,t}$ at period t and $P_{j,t}(i)$ is its corresponding price.

All firms within sector j have access to the same technology, which is given by the CES production function

$$Y_{j,t} = \left(\int_0^1 y_{j,t}(i)^{\frac{\epsilon_j - 1}{\epsilon_j}} di \right)^{\frac{\epsilon_j}{\epsilon_j - 1}}, \tag{2.2.1}$$

where $\epsilon_j > 1$ is the sector specific elasticity of substitution between differentiated inputs. Firms sell the final good to the borrower and the saver at price $P_{j,t}$. Therefore, the per period profits of a firm selling the final good in sector j are given by

$$\Pi^F_{j,t} = P_{j,t} Y_{j,t} - \int_0^1 P_{j,t}(i) y_{j,t}(i) di \tag{2.2.2}$$

Thus, given a sequence of final and intermediate good prices $\left\{ P_{j,\tau}, \{P_{j,\tau}(i)\}_{i \in [0,1]} \right\}_{\tau=0}^{\infty}$, the firm solves the problem of choosing an allocation $\left\{ y_{j,t}(i) \geq 0 \right\}_{i \in [0,1]}$ to maximize (2.2.2) subject to (2.2.1) for each $t \geq 0$.

2.2.2 Producers of Intermediate Goods

As stated before, there is a continuum of intermediate firms indexed by $i \in [0, 1]$ in each sector $j \in \{c, d\}$. The firms operate under monopolistic competition; each firm chooses the price $P_{j,t}(i)$ and variety $y_{j,t}(i)$ to sell to the producers of the final good given the demand for the variety.

Following Calvo (1983), I assume that there is a random variable, $\mathbb{S}_j \sim$ Bernoulli (Φ_j), which governs whether firm i in sector j is able to reset its price in each period. This random variable is i.i.d. across sectors, intermediate firms, and time periods. Thus, firm i within sector j at period t may not reset its price with probability Φ_j.

Intermediate firms use labor as the only input for production. I assume that all firms within a sector have access to the same production technology. Furthermore, to keep the analysis tractable, the production technology is linear in the labor input and given by

$$y_{j,t}(i) = A_j n_{j,t}(i), \tag{2.2.3}$$

where A_j is a constant that measures labor productivity in each sector.[25] The labor market is perfectly competitive and labor is perfectly mobile across sectors; therefore, all intermediate firms pay workers the same nominal wage rate W_t per unit of labor supplied.

Finally, I assume that the intermediate firms are solely owned by the savers and shares of these firms can not be traded.[26] This implies that for any $\tau > t$, the intermediate firms discount the future nominal profits at date τ back to date t with a discount factor given by

$$\Lambda^s_{\tau|t} \equiv \frac{\lambda^s_\tau}{\lambda^s_t} = \beta_s^{\tau-t}\frac{c^s_t}{c^s_\tau}\frac{P_{c,t}}{P_{c,\tau}}, \tag{2.2.4}$$

where λ^s_q is the savers' shadow price of a unit of the nominal asset at period q.

Given the previous specification, for any sequence of nominal wage rates, sector specific outputs, and final good prices, $\left\{W_\tau, \{Y_{j,\tau}, P_{j,\tau}\}_{j\in\{c,d\}}\right\}_{\tau=0}^{\infty}$; an intermediate firm i in sector j that is able to reset its price at period t solves the problem

$$\max_{\{P_{j,t}(i)\}} \mathbb{E}_t\left\{\sum_{k=0}^{\infty}\Phi_j^k\Lambda^s_{t+k|t}\left[P_{j,t}(i)y_{j,t+k|t}(i) - W_{t+k}n_{j,t+k|t}(i)\right]\right\} \tag{2.2.5}$$

$$s.t.$$

$$y_{j,t+k|t}(i) = \left(\frac{P_{j,t+k}}{P_{j,t}(i)}\right)^{\epsilon_j} Y_{j,t+k}$$

$$y_{j,t+k|t}(i) = A_j\, n_{j,t+k|t}(i),$$

where, $\mathbb{E}_t\{\cdot\}$ denotes the firm's expectation given its information set at time t. That is, the intermediate firm chooses the price that maximizes its expected discounted profits given the demand for its variety (the first equality constraint) and its production function (the second equality constraint).

[25] In the current analysis, this constant is set to one in both sectors: $A_d = A_c = 1$.

[26] These two assumption are made to keep the problem tractable.

Note that $y_{j,t+k|t}(i)$ refers to the demand of intermediate good in period $t+k$ provided the firm is unable to reset its price between periods t and $t+k$. It follows that $n_{j,t+k|t}(i)$ is the labor required to produce the amount $y_{j,t+k|t}(i)$.[27]

Finally, note that the total profits for all intermediate firms in sector j at period t are given by

$$\Pi^{I}_{j,t} = \int_0^1 \Big(P_{j,t}(i)y_{j,t}(i) - W_t n_{j,t}(i)\Big)di. \tag{2.2.6}$$

2.2.3 Borrowers and Savers

The economy is populated by two types of agents; borrowers and savers. Let $a \in \{b, s\}$ denote whether an agent is a saver (s) or a borrower (b). The main distinction between these agent types is their time preference discount factor $\beta_s > \beta_b$; savers are more patient than borrowers. Although both agent types are free to borrow or save as desired, in the deterministic steady state with zero inflation the more patient agents effectively lend resources to the less patient agents. Thus the use of the terminology borrowers and savers.

An agent of type a derives utility from consumption of the non-durable (c^a) and durable (d^a) goods and disutility from supplying labor (n^a) according to an additively separable log-linear flow utility function. Therefore, the agent's expected lifetime utility is given by

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_a^t \left[\alpha \log\left(c_t^a\right) + (1-\alpha)\log\left(d_t^a\right) - \nu^a \left(\frac{\left(n_t^a\right)^{(1+\theta)}}{1+\theta} \right) \right] \right\}, \tag{2.2.7}$$

where $\mathbb{E}_t\left\{\cdot\right\}$ denotes the agents conditional expectation given her information set at time t.

In order to finance the consumption of the durable and non-durable goods, agents have four sources of income. First, labor income; the agent can supply labor to the intermediate firms in exchange for the nominal wage rate, W. In doing so, the agent is constraint by her total time endowed, H^a.[28] Second, the share of profits from ownership of the intermediate firms, $\widehat{\Pi}^a$. Third, the market value of the durable holdings (net of depreciation δ) that the agent carries from the previous period. Finally, the interest accrued (given the interest rate R) from holdings of a nominal

[27]The demand for the intermediate variety comes from solving the problem of a firm producing the final good, which is discussed in Section 2.3.1.

[28]I normalize the total time endowment for both agents and make it one; $H^s = H^b = 1$.

risk free bond, B. Thus the nominal budget constraint for the agent at period t is given by

$$P_{c,t}c_t^a + P_{d,t}\left(d_t^a - (1-\delta)\, d_{t-1}^a\right) + R_{t-1}B_{t-1}^a = B_t^a + W_t n_t^a + \widehat{\Pi}_t^a; \tag{2.2.8}$$

where $P_{c,t}$ and $P_{d,t}$ refer to the prices of the non-durable and durable goods.

As stated in Section 2.2.2, I assume that savers are the only owners of the intermediate firms and that shares of these firms can not be endogenously traded. Since the exact distribution of shares among savers is irrelevant, I proceed as if these shares are all equally divided among these agents. Let $S_j^a(i)$ denote the share of firm i in sector j owned by agent of type a, from the previous discussion it is clear that

$$S_j^b(i) \equiv S_j^b = 0, \quad \forall i \in [0,1]$$

$$S_j^s(i) \equiv S_j^s = \frac{1}{\Omega_s}, \ \forall i \in [0,1];$$

where Ω_a denotes the mass of agents of type a.[29] Hence, the share of profits from ownership of the intermediate firms for an agent of type a in period t can be written as

$$\widehat{\Pi}_t^a = \sum_{j\in\{c,d\}} \left(S_j^a\, \Pi_{j,t}^I\right). \tag{2.2.9}$$

Borrowers and savers also differ along another dimension; their access to the nominal bond market. While both agents can save as much as desired using the nominal bond, they are restricted when it comes to borrowing. On one hand, savers face a No-Ponzi condition; the can't infinitely roll-over debt. On the other hand, borrowers face an endogenous credit limit in the form of a collateral constraint, as in Kiyotaki and Moore (1997). The relatively less patient agents might be more likely to default on debt or to refuse to honor the terms of the contract. Thus the provision of collateral provides enough incentive for them to endogenously choose to repay the debt according to the contract. The particular form of this credit constraint is given by

$$R_t B_t^b \leq (1-\chi)\,(1-\delta)\ d_t^b\, \mathbb{E}_t\left\{P_{d,t+1}\right\}. \tag{2.2.10}$$

[29]The total mass of the population in the economy is normalized to be one; $\Omega_s + \Omega_b = 1$.

The left side of the inequality is just the amount the borrower must repay, principal plus interest, in period $t+1$. The right hand side of the inequality is the collateral requirement; the agent is allowed to pledge only a fraction $(1-\chi)$ of her durable holdings as collateral. The value of the collateral is determined by the expected price of the durable good in the repayment period $t+1$. Therefore, the constraint simply requires that the repayment value of a given loan does not exceed the collateral's expected value.

All in all, given initial wealth holdings $\left(d_{-1}^{a}, R_{-1}B_{-1}^{a}\right)$, prices $\{P_{c,t}, P_{d,t}, W_t, R_{t-1}\}_{t=0}^{\infty}$, and intermediate firms' profits $\left\{\Pi_{c,t}^{I}, \Pi_{d,t}^{I}\right\}_{t=0}^{\infty}$; the problem of an agent of type $a \in \{b, s\}$ is to choose an allocation $\{c_t^a, d_t^a, n_t^a, B_t^a\}_{t=0}^{\infty}$, with $c_t^a, d_t^a \geq 0$ and $n_t^a \in [0, H^a]$, to maximize (2.2.7) subject to (2.2.8). Additionally, if the agent is of type borrower, the problem has the additional credit constraint (2.2.10).

2.2.4 Monetary Authority

I assume that monetary policy is conducted via an interest rate rule of the type proposed by Taylor (1993). Given a nominal interest rate target $\tilde{R}$ and inflation target $\tilde{\pi}$, the monetary authority follows the interest rate rule given by

$$\frac{R_t}{\tilde{R}} = \left(\frac{\pi_t}{\tilde{\pi}}\right)^{\phi_\pi} z_t, \tag{2.2.11}$$

where R_t is the interest rate on nominal bond contracts and π_t is a composite inflation index that weights the inflation in the durable $\left(\pi_{d,t} \equiv \frac{P_{d,t}}{P_{d,t-1}}\right)$ and non-durable $\left(\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}\right)$ sectors according to $\pi_t = \pi_{c,t}^{\kappa}\pi_{d,t}^{1-\kappa}$. The parameter $\kappa \in (0,1)$ controls the relative weight that the monetary authority gives to each of the two sectors in the economy.[30]

Furthermore, I assume that the interest rule follows the Taylor principle; $\phi_\pi > 1$. I impose this assumption given that it is standard in the literature; where it is used for two main reasons. First, the empirical evidence suggests that the failure of the monetary authority to follow this principle has led to episodes of greater macroeconomic instability in the US.[31] Second, the Taylor principle usually arises as a necessary and sufficient condition for the existence of a unique stable solution

[30] For the baseline model, κ is set to 0.5 so that the monetary authority weights the two sectors equally.

[31] See Taylor (1999) and Clarida, Galí and Gertler (2000).

for some infinite horizon New Keynesian forward looking models.[32]

The variable z_t is an exogenous stochastic component that captures the unanticipated actions of the monetary authority.[33] It is assumed that such process follows an AR(1) of the form

$$\log(z_t) = \rho \log(z_{t-1}) + \sigma^z \epsilon_t^z, \tag{2.2.12}$$

where $\rho \in (0, 1)$ is the autocorrelation coefficient which regulates the degree of persistence of the process, $\epsilon_t^z \sim \mathrm{WN}(0, 1)$ is the underlying monetary policy shock, and σ^z is a parameter that controls the standard deviation of the underlying shock.

2.3 Solution

This section discusses the solution to the model presented in Section 4.3. I start by defining some important aggregate variables. Next, I define the equilibrium concept used in solving the model and provide a partial characterization of this solution. Finally, I consider the special case of the deterministic steady state, which provides some useful insights about the behavior of the agents in the model.

2.3.1 Aggregate Variables

The solution to the problem of the firms producing the final sectorial goods yields the demand for intermediate varieties, which is given by

$$y_{j,t}(i) = \left(\frac{P_{j,t}}{P_{j,t}(i)} \right)^{\epsilon_j} Y_{j,t}. \tag{2.3.1}$$

In setting their prices, the intermediate firms take this demand as given; which is captured by the first constraint in the intermediate firms' problem (2.2.5). Additionally, given the perfectly competitive nature of the final good's market, the zero-profit condition implies that the aggregate price index of each sector satisfies

$$P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\epsilon_j} di \right)^{\frac{1}{1-\epsilon_j}}. \tag{2.3.2}$$

[32] See Woodford (2001) and Woodford (2003).

[33] For various interpretations of this monetary policy shock, refer to Christiano, Eichenbaum and Evans (1999).

Another important aggregate variable, total labor employed, can be obtained by considering the intermediate firms' problem. Aggregating over the labor employed by each individual firm in sector j we have

$$N_{j,t} \equiv \int_0^1 n_{j,t}(i)\,di = \frac{Y_{j,t}}{A_j}\int_0^1 \left(\frac{P_{j,t}}{P_{j,t}(i)}\right)^{\epsilon_j} di. \tag{2.3.3}$$

The second equality follows from manipulating and combining the two constraints in the intermediate firms' problem (2.2.5). Equation (2.3.3) can be interpreted as stating that the final good producers and the intermediate firms can effectively be thought of as a representative firm in each sector. This representative firm produces the final sectorial good using aggregate labor according to a production function of the form $Y_{j,t} = \tilde{A}_j \cdot N_{j,t}$; where $\tilde{A}_j \equiv A_j \cdot \Delta^{-1}_{P_j,t}$ is the effective aggregate labor productivity. In light of this observation, and for the remainder of the chapter, equation (2.3.3) will be called the aggregate production function. Additionally, the representative firm's profits are equivalent to the profits made by the intermediate firms since

$$\Pi^I_{j,t} \equiv \int_0^1 \Big(P_{j,t}(i)y_{j,t}(i) - W_t n_{j,t}(i)\Big)di = P_{j,t}Y_{j,t} - W_t N_{j,t}. \tag{2.3.4}$$

Note that the effective aggregate labor productivity is a scaled version of the actual sectorial labor productivity, A_j. The scaling factor, $\Delta_{P_j,t}$, is a measure of the intermediate firm's sectorial price dispersion. It is defined as

$$\Delta_{P_j,t} \equiv \int_0^1 \left(\frac{P_{j,t}}{P_{j,t}(i)}\right)^{\epsilon_j} di. \tag{2.3.5}$$

Thus $\Delta_{P_j,t}$ is an indicator of the extent to which the individual firms' prices in sector j deviate relative to the aggregate sectorial price level. Note that as the price dispersion gets large, the effective aggregate labor productivity decreases; to produce a given amount of output the intermediate firms in sector j need to hire more labor.

Finally, the last aggregate variable of importance is sectorial price inflation $\pi_{j,t}$. As customary in the literature, I define it as $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}},\ \ j \in \{c,d\}$. Note that from the relationship between the sectorial price index and the intermediate firm's prices given in (2.3.2), the sectorial inflation index can be written as

$$\pi_{j,t}^{1-\epsilon_j} = \int_0^1 \left(\frac{P_{j,t}(i)}{P_{j,t-1}} \right)^{1-\epsilon_j} di. \tag{2.3.6}$$

That is, inflation in each sector is a function of how, on average, the prices of each individual firm in sector j deviate from the aggregate sectorial price index in the previous period.

The aggregate price dispersion and the sectorial inflation, equations (2.3.5) and (2.3.6), can be written in a more intuitive, easy to interpret, form. To this end, note that any two intermediate firms in sector j face an identical problem and let $\Upsilon_{j,t} \subset [0,1]$ refer to the set of firms in sector j that are able to reset their price in period t. It is obvious then tat the price chosen by any two resetting firms is the same; $P_{j,t}(i) \equiv P^*_{j,t}$, $\forall\, i \in \Upsilon_{j,t}$. Thus these two equations can be written as

$$\begin{aligned}
\Delta_{P_j,t} &= \left(\frac{P_{j,t}}{P^*_{j,t}} \right)^{\epsilon_j} \int_{\Upsilon_{j,t}} 1\, di + \pi_{j,t}^{\epsilon_j} \int_0^1 \left(\frac{P_{j,t-1}}{P_{j,t-1}(i)} \right)^{\epsilon_j} \mathbb{1}_{\{i \in [0,1] \setminus \Upsilon_{j,t}\}}\, di \\
&= \left(\frac{P_{j,t}}{P^*_{j,t}} \right)^{\epsilon_j} \int_{\Upsilon_{j,t}} 1\, di + \pi_{j,t}^{\epsilon_j} \int_0^1 \left(\frac{P_{j,t-1}}{P_{j,t-1}(i)} \right)^{\epsilon_j} di \int_{[0,1] \setminus \Upsilon_{j,t}} 1\, di \\
&= \left(\frac{P_{j,t}}{P^*_{j,t}} \right)^{\epsilon_j} (1 - \Phi_j) + \pi_{j,t}^{\epsilon_j} \Delta_{P_j,t-1} \Phi_j
\end{aligned} \tag{2.3.7}$$

$$\begin{aligned}
\pi_{j,t}^{1-\epsilon_j} &= \left(\frac{P^*_{j,t}}{P_{j,t-1}} \right)^{1-\epsilon_j} \int_{\Upsilon_{j,t}} 1\, di + \int_0^1 \left(\frac{P_{j,t-1}(i)}{P_{j,t-1}} \right)^{1-\epsilon_j} \mathbb{1}_{\{i \in [0,1] \setminus \Upsilon_{j,t}\}}\, di \\
&= \left(\frac{P^*_{j,t}}{P_{j,t}} \right)^{1-\epsilon_j} \pi_{j,t}^{1-\epsilon_j} \int_{\Upsilon_{j,t}} 1\, di + \int_0^1 \left(\frac{P_{j,t-1}(i)}{P_{j,t-1}} \right)^{1-\epsilon_j} di \int_{[0,1] \setminus \Upsilon_{j,t}} 1\, di \\
&= \left(\frac{P^*_{j,t}}{P_{j,t}} \right)^{1-\epsilon_j} \pi_{j,t}^{1-\epsilon_j} (1 - \Phi_j) + \Phi_j
\end{aligned} \tag{2.3.8}$$

In deriving equations (2.3.7) and (2.3.8), the i.i.d. assumption of the random variable $\mathbb{S}_j \sim$ Bernoulli (Φ_j) is crucial as it allows to write the integral in the first line as the product of integrals in the second line. In addition, although the set of firms that are able to reset their price at any time period is random, the measure of such set is deterministic and equal to equal to $1 - \Phi_j$; which is used to obtain the final line of the derivations.

These two expressions completely determine the aggregate price dynamics of the economy. Start with a known distribution of prices in the intermediate firms; which is summarized by the price dispersion $\Delta_{P_j,t-1}$. From the optimal behavior of the intermediate firms in the current period, equation (2.3.8) pins down the sectorial inflation levels. Given the sectorial inflation, equation (2.3.7) then determines the new price distribution of the intermediate firms within sector j; which again is summarized by the statistic $\Delta_{P_j,t}$.

2.3.2 Equilibrium Concept

Define real bond holdings $b_t^a \equiv \frac{B_t^a}{P_{c,t}}$, relative intermediate firms' prices $p_{j,t}^* \equiv \frac{P_{j,t}^*}{P_{j,t}}$, real intermediate firms' profits $\overline{\Pi}_{c,t}^I \equiv \frac{\Pi_{j,t}^I}{P_{j,t}}$, relative price of the durable good $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$, and real wage rates $w_{j,t} \equiv \frac{W_t}{P_{j,t}}$. Given initial real wealth holdings for the agents $\left\{d_{-1}^a, R_{-1}b_{-1}^a\right\}_{a\in\{b,s\}}$ and sectorial price dispersion $\{\Delta_{P_c,-1}, \Delta_{P_d,-1}\}$, an equilibrium for this economy is defined as sequences of

(i) <u>Agents' allocation:</u>	$\mathbf{A} \equiv \left\{\left\{c_t^a, d_t^a, n_t^a, b_t^a\right\}_{t=0}^{\infty}\right\}_{a\in\{b,s\}}$
(ii) <u>Intermediate firms' prices:</u>	$\mathbf{P}^I \equiv \left\{p_{c,t}^*, p_{d,t}^*\right\}_{t=0}^{\infty}$
(iii) <u>Aggregate Output:</u>	$\mathbf{O} \equiv \{Y_{c,t}, Y_{d,t}, N_{c,t}, N_{d,t}\}_{t=0}^{\infty}$
(iv) <u>Aggregate Prices:</u>	$\mathbf{P} \equiv \{q_t, \pi_{c,t}, \pi_{d,t}, \Delta_{P_c,t}, \Delta_{P_d,t}, w_{c,t}, w_{d,t}, R_t\}_{t=0}^{\infty}$
(v) <u>Intermediate firms' profits:</u>	$\mathbf{\Pi}^I \equiv \left\{\overline{\Pi}_{c,t}^I, \overline{\Pi}_{d,t}^I\right\}_{t=0}^{\infty}$

such that the following hold:

1. **Monetary Policy**

   - Given the exogenous process (2.2.12), the aggregate price sequence **P** satisfies the monetary policy rule (2.2.11).

2. **Agents' optimization**

   - Given aggregate prices and profits (**P** and $\mathbf{\Pi}^I$), the allocation **A** is a solution to the borrowers' and savers' problems.

- Given aggregate output and prices (**O** and **P**), the prices $\mathbf{P}^I$ are a solution to the intermediate firms' problems.

3. **Aggregate Variables and Price Dynamics**

- The aggregate output and prices (**O** and **P**) satisfy the aggregate production function (2.3.3).

- Given aggregate output and prices (**O** and **P**), the intermediate firms' profits $\mathbf{\Pi}^I$ satisfy (2.3.4).

- The intermediate firms' prices and the aggregate price sequences ($\mathbf{P}^I$ and **P**) satisfy the the price dynamics equations (2.3.7) and (2.3.8).

4. **Market Clearing**

- The agents' allocation and aggregate output sequences(**A** and **O**) satisfy the market clearing conditions

  (a) Final non-durable good market: $Y_{c,t} = \Omega_b c_t^b + \Omega_s c_t^s$

  (b) Final durable good market: $Y_{d,t} = \sum_{a \in \{s,b\}} \Omega_a \left(d_t^a + (1-\delta) d_{t-1}^a \right)$

  (c) Nominal debt market: $\Omega_b B_t^b + \Omega_s B_t^s = 0$

  (d) Labor market: $N_{c,t} + N_{d,t} = \Omega_b n_t^b + \Omega_s n_t^s$

2.3.3 Equilibrium Characterization

Given the assumed functional forms for technology and preferences, the FOC's are both necessary and sufficient to characterize the firms' and agents' problems.

Firms

Consider first an intermediate firm's problem. From the FOC's, a resetting firm's optimal price is given by

$$p_{j,t}^* = \left(\frac{\epsilon_j}{\epsilon_j - 1} \right) \frac{\mathrm{MC}_{j,t}}{\widehat{\mathrm{MR}}_{j,t}}; \tag{2.3.9}$$

where $\mathrm{MC}_{j,t} \equiv \mathbb{E}_t \left[\sum_{k=0}^{\infty} \Gamma_{j,t+k}\, \omega_{j,t+k} \left(q_{t+k}^{\mathbb{1}_{j=d}} \right) \right]$ is the expected discounted real marginal cost of all future periods, $\widehat{\mathrm{MR}}_{j,t} \equiv \mathbb{E}_t \left[\sum_{k=0}^{\infty} \Gamma_{j,t+k} \left(\frac{P_{j,t}}{P_{j,t+k}} \right) \left(q_{t+k}^{\mathbb{1}_{j=d}} \right) \right]$ is the expected discounted real marginal revenue if the firm were to sell its variety at the current aggregate price level in all future periods, and $\Gamma_{j,t+k} \equiv (\Phi_j \beta_s)^k \left(\frac{Y_{j,t+k}}{c_{t+k}^s} \right) \left(\frac{P_{j,t+k}}{P_{j,t}} \right)^{\epsilon_j}$ plays the role of an effective real discount factor.

To understand the discounted marginal cost $\mathrm{MC}_{j,t}$, note that given the assumption of a linear technology with labor productivity $A_j = 1$, producing an additional unit of sectorial output at period $t+k$ requires exactly one additional unit of labor. Thus the sectorial real marginal cost is the real wage rate $w_{j,t+k}$. As for the discounted marginal benefit $\widehat{\mathrm{MR}}_{j,t}$, if the firm sells a unit of the variety at period $t+k$ and charges the current aggregate sectorial price level, it raises a real revenue of $\frac{P_{j,t}}{P_{j,t+k}}$. If the firm decides to charge the price $p_{j,t}^*$ instead, the real revenue is given by $\frac{p_{j,t}^*}{P_{j,t+k}}$ and the expected discounted marginal revenue for this case can be written as $\mathrm{MR}_{j,t} \equiv p_{j,t}^* \cdot \widehat{\mathrm{MR}}_{j,t}$.[34] Therefore, equation (2.3.9) states that a resetting intermediate firm optimally chooses its price so that its the expected discounted real marginal revenue has a mark-up of $\left(\frac{\epsilon_j}{\epsilon_j - 1} > 1 \right)$ over its expected discounted real marginal cost.

From the previous discussion, it is clear that there are two driving forces behind the pricing behavior of the intermediate firms; the expected changes in future real wage rates and the expected changes of future aggregate prices. If firms expect large future real wages, they will choose to set a higher price in the current period. However, for any $k > 0$, the expected future real wage $\omega_{j,t+k}$ does not only affect the current price chosen by the resetting firms; it affects all prices chosen between periods t and $t+k$. The intermediate firms choose to change prices "smoothly"; rather than having a one time change, firms optimally adjust the prices of all the periods between the current period and the period in which the deviation of the labor cost is expected. Therefore, only unanticipated deviations of labor costs create sudden changes in prices.

Similarly, firms adjust their optimal price in response to expectations of future aggregate prices. Larger future inflation provides an incentive for firms to increase the current price; firms value more current revenue given the inflationary expectations. Again, the expected future aggregate price level $P_{j,t+k}$ has an effect on all prices chosen between periods t and $t+k$; only unanticipated changes in inflation create sudden changes in firms' prices.

[34]The factor of q_t for the durable sector in both the $\mathrm{MC}_{j,t}$ and $\widehat{\mathrm{MR}}_{j,t}$ is just to express these quantities in units of the non-durable good.

Given the pricing decision of the intermediate firms, the impact on the aggregate prices (inflation) is given by equation (2.3.8). Since $\epsilon_j > 1$, the current sectorial inflation is an increasing function of the price chosen by the intermediate firms. Not surprisingly, if intermediate firms choose a higher price the current sectorial aggregate price increases, which in turn has an inflationary effect.

Finally, given the sectorial inflation and the pricing decision of the intermediate firms, the economy's price distribution is summarized by the resulting price dispersion as given by equation (2.3.7). Intuitively, the price dispersion is the result of a combination of two effects. On one hand, the resetting firms (with mass $1 - \Phi$) affect the price dispersion given their chosen optimal price $p^*_{j,t}$. On the other hand, the non-resetting firms (with mass Φ) affect the price dispersion to the extent that the current aggregate price differs from the previous period prices; thus their contribution is driven by the sectorial inflation $\pi_{j,t}$.

Putting together all the previous results, the production sector of the economy can be characterized by the following set of dynamic equations:

$$\mathrm{MC}_{j,t} = \omega_{c,t} \left(\frac{Y_{j,t}}{c^s_t} \right) + \Phi_j \beta_s \mathbb{E}_t \left\{ \pi^{\epsilon_j}_{j,t+1} \, \mathrm{MC}_{j,t+1} \right\} \tag{2.3.10}$$

$$\widehat{\mathrm{MB}}_{j,t} = q_t^{\mathbb{1}_{j=d}} \left(\frac{Y_{j,t}}{c^s_t} \right) + \Phi_j \beta_s \mathbb{E}_t \left\{ \pi^{\epsilon_j - 1}_{j,t+1} \, \widehat{\mathrm{MB}}_{j,t+1} \right\}, \tag{2.3.11}$$

$$\pi_{j,t} = \left[\frac{1}{\Phi_j} - \left(\frac{1 - \Phi_j}{\Phi_j} \right) \left(\frac{\epsilon_j - 1}{\epsilon_j} \right)^{\epsilon_j - 1} \left(\frac{\widehat{\mathrm{MB}}_{j,t}}{\mathrm{MC}_{j,t}} \right)^{\epsilon_j - 1} \right], \tag{2.3.12}$$

$$\Delta_{P_j,t} = \left(\frac{\epsilon_j - 1}{\epsilon_j} \right)^{\epsilon_j} \left(\frac{\widehat{\mathrm{MB}}_{j,t}}{\mathrm{MC}_{j,t}} \right)^{\epsilon_j} (1 - \Phi_j) + \pi^{\epsilon_j}_{j,t} \Delta_{P_j,t-1} \Phi_j. \tag{2.3.13}$$

$$q_t = \left(\frac{\pi_{d,t}}{\pi_{c,t}} \right) q_{t-1}, \ \ \forall t \geq 0; \tag{2.3.14}$$

where the last one follows directly from the definition of q_t.

Borrowers and Savers

Before analyzing the optimal behavior of the agents, note that the nominal budget constraint (2.2.8) can be written in real (non-durable) units as

$$c_t^a + q_t\left(d_t^a - (1-\delta)\, d_{t-1}^a\right) + \frac{R_{t-1}}{\pi_{c,t}} b_{t-1}^a = b_t^a + \omega_{c,t} n_t^a + \overline{\Pi}_t^a; \tag{2.3.15}$$

where the last term stands for the share of real profits that agent a gets from the intermediate firms, which is given by $\overline{\Pi}_t^a \equiv \sum_{j\in\{c,d\}} S_j^a \left(q_t^{\mathbb{1}_{j=d}} Y_{j,t} - \omega_{c,t} N_{j,t}\right)$.

Consider now the intra-temporal tradeoff that agent a faces between consumption and leisure given by

$$\left(\frac{\alpha}{c_t^a}\right)\omega_{c,t} = \nu^a \left(n_t^a\right)^\theta, \;\; \forall t \geq 0. \tag{2.3.16}$$

This is a standard condition which states that the agent optimally equates the marginal benefit of labor supplied (left hand side) to its marginal cost (right hand side).

Consider next the inter-temporal tradeoffs faced by the agent. Since the agent can transfer wealth via the durable good and the nominal bond, there are two inter-temporal conditions that the agent's optimal allocation must satisfy. The condition related to the durable good is

$$\alpha\left(c_t^a\right)^{-1} q_t = \frac{(1-\alpha)\left(d_t^a\right)^{-1} + \beta_a (1-\delta)\,\mathbb{E}_t\left\{\alpha\left(c_{t+1}^a\right)^{-1} q_{t+1}\right\}}{1-(1-\chi)(1-\delta)\,\zeta_t^a \mathbb{E}_t\left\{\pi_{d,t+1}\right\}}, \tag{2.3.17}$$

where $\zeta_t^a \equiv \psi_t^a \left(\lambda_t^a\right)^{-1}$, ψ_t^a is the multiplier on the collateral credit constraint (2.2.10), and $\lambda_t^a = \alpha\left(c_t^a P_{c,t}\right)^{-1}$ is the multiplier on the budget constraint (2.2.8).[35] This condition just states that the agent optimally equates the marginal cost and marginal benefit of durable consumption. The marginal cost is given by the forgone consumption of the non-durable good in the current period; valued at $\alpha\left(c_t^a\right)^{-1} q_t$. The marginal benefit is composed of three terms. First, the marginal value of current durable consumption $(1-\alpha)\left(d_t^a\right)^{-1}$. Second, the marginal value of the wealth transferred to the next period via the durable good, $\beta_a (1-\delta)\,\mathbb{E}_t\left\{\alpha\left(c_{t+1}^a\right)^{-1} q_{t+1}\right\}$. Lastly, the marginal value of relaxing the borrowing constraint; the extra borrowing allows the agent to consume $(1-\chi)(1-\delta)\,\zeta_t^a \mathbb{E}_t\left\{\pi_{d,t+1}\right\}$ additional units of non-durable good, each marginally valued at $\alpha\left(c_t^a\right)^{-1} q_t$. Note that the third component of the marginal benefit of durable consumption is only relevant if the credit constraint is binding (i.e. only if $\zeta_t^a > 0$).

[35] Since the savers don't face a collateral constraint, $\psi_t^s = 0, \;\; \forall\, t \geq 0$.

The condition summarizing the inter-temporal tradeoff of the nominal bond is given by

$$1 = R_t \left(\zeta_t^a + \beta_a \mathbb{E}_t \left\{ \frac{c_t^a}{c_{t+1}^a} \frac{1}{\pi_{c,t+1}} \right\} \right). \tag{2.3.18}$$

On one had, the marginal benefit of nominal debt is reflected on the increase in current consumption by $P_{c,t}^{-1}$ units; each marginally valued at $\alpha \left(c_t^a\right)^{-1}$. On the other hand, the marginal cost of nominal debt has two components. First, the agent must repay the borrowed amount in the next period, thus foregoing future consumption which is valued at $\alpha \beta_a R_t \mathbb{E}_t \left(P_{c,t+1} c_{t+1}^a\right)^{-1}$. Second, to the extent that the agent is credit constrained, the additional borrowing requires an increase in collateral holdings; the agent needs to give up current resources to increase the collateral. This additional cost is captured by the term $R_t \psi_t^a$. Equating these costs to the marginal benefit and rearranging one gets (2.3.18).

Given the forward looking nature of the expressions (2.3.17) and (2.3.18), transversality conditions are needed as part of the solution characterization. The general form of the transversality condition for agent a is

$$\lim_{t \to \infty} \mathbb{E}_0 \left\{ \alpha \beta_a^t \left(q_t \left(1 - \delta\right) \frac{d_{t-1}^a}{c_t^a} + \frac{R_{t-1}}{\pi_{c,t}^a} \frac{b_{t-1}^a}{c_t^a} \right) \right\} = 0, \tag{2.3.19}$$

where $b_t^a = B_t^a \left(P_{c,t}\right)^{-1}$ is the real debt holdings of agent a in terms of the non-durable good. Intuitively, these transversality conditions impose that the expected present value of the agents' wealth is zero in the limit. If this was not the case, agents would accumulate infinite amounts of wealth.

Finally, the characterization of the agent's behavior is completed via the complementary slackness conditions. Given that the nominal budget constraint (2.3.15) holds with equality for both agents, the multiplier on this constraint (λ_t^a) will be strictly positive for all periods t. Since the saver faces no credit constraint, the only relevant complementary slackness condition is the one imposed on the borrower

$$\psi_t^b \left[R_t b_t^b - \left(1 - \chi\right) \left(1 - \delta\right) q_t d_t^b \mathbb{E}_t \left\{ \pi_{d,t+1} \right\} \right] = 0. \tag{2.3.20}$$

Equilibrium Characterization

Assume the the initial state $\left(R_{-1}b_{-1}^s, R_{-1}b_{-1}^b, d_{-1}^s, d_{-1}^b, \Delta_{P_c,-1}, \Delta_{P_d,-1}, q_{-1}\right)$ is given. The equilibrium is characterized by an allocation $\left\{\{c_t^a, d_t^a, n_t^a, b_t^a\}_{a\in\{b,s\}}, \{Y_{j,t}, N_{j,t}\}_{j\in\{c,d\}}, \zeta_t^b\right\}_{t=0}^{\infty}$ and price $\left\{\left\{\mathrm{MC}_{j,t}, \widehat{\mathrm{MB}}_{j,t}, \pi_{j,t}, \Delta_{P_j,t}, \omega_{j,t},\right\}_{j\in\{c,d\}}, R_t, q_t\right\}_{t=0}^{\infty}$ sequences that satisfy

1. The monetary policy rule (2.2.11).
2. The aggregate production function (2.3.3).
3. The intermediate firms' pricing behavior (2.3.10) and (2.3.11).
4. The aggregate price dynamics (2.3.12), (2.3.13), and (2.3.14)
5. The agents' optimal behavior (2.3.15),(2.3.16), (2.3.17), (2.3.18), (2.3.19), and (2.3.20).
6. The market clearing conditions listed in Section 2.3.2.

2.3.4 Deterministic Steady State

The deterministic steady state of the model is defined as the set

$$\tilde{S} \equiv \left\{\left\{\tilde{c}^a, \tilde{d}^a, \tilde{n}^a, \tilde{b}^a\right\}_{a\in\{b,s\}}, \tilde{\zeta}^b, \left\{\widetilde{Y}_j, \widetilde{N}_j, \widetilde{\mathrm{MC}}_j, \widetilde{\mathrm{MB}}_j, \widetilde{\pi}_j, \widetilde{\Delta}_{P_j}, \widetilde{\omega}_j\right\}_{j\in\{c,d\}}, \widetilde{R}, \widetilde{q}\right\};$$

where $\tilde{x} \in \tilde{S}$ satisfies $x_t = x_{t+1} = \tilde{x}$, $\forall t \geq 0$. That is, in the model's steady state all variables are constant over time. In the analysis that follows I restrict attention to the deterministic steady state with zero sectorial inflation; $\tilde{\pi}_d = \tilde{\pi}_c = 1$.

The current chapter does not provide formal proofs for the existence and uniqueness of such steady state; instead, it relies on the existing literature.[36] In models with heterogenous discount rates and perfect financial markets, the existence of a steady state is not guaranteed. If the savers' discount factor determines the market interest rate, the borrowers' consumption could not remain constant and it would asymptotically decrease over time. If the borrowers' discount factor pins down the interest rate, the savers' consumption would be asymptotically increasing over time. In the current model, the collateral constraint allows one to get around this difficulty; it creates a wedge between

[36]The determinacy of the steady state in models with heterogeneous agents has been studied by authors such as Becker (1980) and Becker and Foias (1987) among others.

the savers' and borrowers' effective interest rates thus ensuring both agents can sustain a constant consumption over time. Additionally, the heterogeneity in the discount factor ($\beta_s > \beta_b$) guarantees that this steady state is independent of the initial wealth distribution of the agents. Therefore, the borrowing constraint and the discount factor heterogeneity ensure the existence and uniqueness of the deterministic steady state with zero inflation.

In order to characterize the steady state, consider first the production sector. Given the assumption of zero inflation, intermediate firms behave as in the case of monopolistic competition with perfectly flexible pricing. There are two immediate implications for such behavior. First, there is no price dispersion; $\widetilde{\Delta}_{P_c} = \widetilde{\Delta}_{P_d} = 1$. Second, firms set their prices to ensure that their marginal revenue is equal to a constant markup of $\frac{\epsilon_j}{\epsilon_j - 1}$ over their marginal production cost. Recall that given the linear production function, the marginal cost of each unit is just the wage rate; $\mathrm{MC}_j = \omega_j$. Additionally, the marginal revenue is given by the price at which firms sell the good (which in steady state is normalized to 1); thus $\widetilde{\mathrm{MB}}_j = 1$. Hence the firms' optimal price setting behavior of equating the marginal revenue to a markup over the marginal cost implies that

$$\widetilde{\omega}_j = 1 - \frac{1}{\epsilon_j}, \quad j \in \{c, d\}. \tag{2.3.21}$$

That is, the steady state sectorial wage rate is equal to the inverse of the firms' markup. In turn, the relationship between the sectorial wage rates determines the relative price of the durable good. To see this, note that the perfect mobility of labor implies that the nominal wage rate must be the same across both sectors in the economy; $\widetilde{W} = \widetilde{P}_j \cdot \widetilde{\omega}_j$ for $j \in \{c, d\}$. Therefore

$$\tilde{q} = \frac{\tilde{\omega}_c}{\tilde{\omega}_d} = \frac{(\epsilon_c - 1)\,\epsilon_d}{(\epsilon_d - 1)\,\epsilon_c}. \tag{2.3.22}$$

The behavior of the borrowers and savers determines the remaining steady state variables. From the inter-temporal tradeoff for the nominal bond, equation (2.3.18), the steady state interest rate is pinned down by the savers' preference discount factor; $\tilde{R} = (\beta_s)^{-1}$. In turn, this implies that the borrowing constraint is binding for the borrowers in steady state; $\tilde{\zeta}^b = \beta_s - \beta_b > 0$. To understand the meaning of the value of $\tilde{\zeta}^b$, consider the following thought experiment. Suppose the collateral constraint is relaxed by increasing the fraction of the durable good that can be pledged as collateral (i.e. a decrease in χ). Furthermore, assume this is done in such a way so that the borrowers can increase their real debt holdings by β_s units without changing their holdings of the durable good; the borrowers can now derive an additional utility of $\beta_s \tilde{\lambda}_t^b$ in the current period. However, the value of the debt that the borrowers must repay in the next period is equal to $\widetilde{R} \cdot \beta_s = 1$; foregoing this unit of consumption implies a disutility of $\beta_b \tilde{\lambda}_t^b$.[37] Therefore, the value that the borrowers attach to relaxing the collateral constraint is given by $\tilde{\psi}_t^b = (\beta_s - \beta_b) \tilde{\lambda}_t^b$. The desired result follows since $\tilde{\zeta}^b \equiv \frac{\tilde{\psi}_t^b}{\tilde{\lambda}_t^b}$. Finally, since the borrowing constraint is binding, this implies that the ratio of debt-to-durable holdings for the borrower is fixed and given by

$$\frac{\tilde{b}^b}{\tilde{d}^b} = (1-\chi)(1-\delta) \frac{\tilde{q}}{\widetilde{R}}. \tag{2.3.23}$$

The borrowers' and savers' consumption allocations are obtained from the inter-temporal durable holdings condition and the budget constraint, equations (2.3.17) and (2.3.15), and are given by

$$\frac{\tilde{d}^s}{\tilde{c}^s} = \left(\frac{1-\alpha}{\tilde{q}\alpha}\right) \left(\frac{1}{1-\beta_s(1-\delta)}\right) \text{ and} \tag{2.3.24}$$

$$\frac{\tilde{d}^b}{\tilde{c}^b} = \left(\frac{1-\alpha}{\tilde{q}\alpha}\right) \left(\frac{1}{1-\beta_s(1-\delta)+\chi(1-\delta)\tilde{\zeta}^b}\right). \tag{2.3.25}$$

$$\frac{\tilde{n}^b}{\tilde{c}^b} = \frac{1+\tilde{q}\left(1-\chi(1-\delta)-(1-\chi)(1-\delta)\beta_s\right)\dfrac{\tilde{d}^b}{\tilde{c}^b}}{\tilde{\omega}_c}. \tag{2.3.26}$$

It is clear from these expressions that the ratio of durable-to-nondurable consumption is smaller for the borrowers; $\frac{\tilde{d}^b}{\tilde{c}^b} \leq \frac{\tilde{d}^s}{\tilde{c}^s}$, with the inequality holding strictly for $\chi > 0$ and $\delta < 1$.

To better understand the durable-to-nondurable consumption ratios, it is useful to look at two extreme cases; the case of full depreciation $(\delta = 1)$ and when the durable holdings can be fully pledged as collateral $(\chi = 0)$. Consider firs the full depreciation case; the durable good effectively looses its "storability" property. Therefore, it is no different than the non-durable good and the ratio between the two is completely determined by their relative, static, marginal utilities. Given the

[37]Note that the shadow value of income is given by $\lambda_t^b = \beta_b^t \alpha \left(c_t^b\right)^{-1}$. However, in steady state we have $c_t^b = \tilde{c}^b \; \forall t$; so that $\tilde{\lambda}_{t+1}^b = \beta_b^t \tilde{\lambda}_t^b$.

agents are homogeneous in this dimension, the ratio of durable-to-nondurable consumption is the same for the borrowers and the savers.

The case where the durable good can be fully pledged as collateral is useful to illustrate the effect of the borrowing constraint on the borrowers consumption ratio. When a fraction χ of every unit of the non-depreciated durable good can't be pledged as collateral, the borrowers face an additional cost; the foregone value of these χ units whose next best use would be as collateral. Thus the term $\chi(1-\delta)\tilde{\zeta}^b$ on the denominator of (2.3.25) can be interpreted as the cost associated with durable holdings that can't be pledged as collateral.[38] Given that the borrowers face this additional cost for holding durable good, the durable-to-nondurable ratio is smaller for the borrowers compared to the savers. For the case in which all of the durable good holdings can be pledged as collateral $(\chi = 0)$, the borrowing constraint does not imply this additional cost for durable holdings; thus the ratio of durables to non-durables is the same for both agents despite the borrowers facing the additional constraint.

Lastly, equation (2.3.26) presents the ratio of labor-to-nondurable consumption for the borrowers. This ratio is completely determined by the budget constraint; borrowers must use labor income to finance the portion of durable and non-durable consumption that can't be covered via their wealth (durable savings net of debt obligations).[39]

The relationship between the borrowers' and savers' consumption is determined via the market clearing conditions. In particular, the ratio of non-durable consumption between the agents is given by

$$\frac{\tilde{c}^s}{\tilde{c}^b} = \left[(1-\Omega_b)\left(1+\delta\frac{\tilde{d}^s}{\tilde{c}^s}\right)\right]^{-1}\left[\frac{\tilde{n}}{\tilde{c}^b} - \Omega_b\left(1+\delta\frac{\tilde{d}^b}{\tilde{c}^b}\right)\right]. \tag{2.3.27}$$

which is obtained under the assumptions that $\Omega_s + \Omega_b = 1$ and $\tilde{n}^s = \tilde{n}^b \equiv \tilde{n}$.[40] Since the ratio of durable-to-nondurable consumption is smaller for the borrowers $\left(\frac{\tilde{d}^b}{\tilde{c}^b} \leq \frac{\tilde{d}^s}{\tilde{c}^s}\right)$, and as long as $\frac{\tilde{n}}{\tilde{c}^b}$ is sufficiently large, savers' non-durable consumption is larger than that of the borrowers.[41] Intuitively, borrowers accumulate a lower level of wealth as they are relatively more impatient; consumption

[38] Recall that $\tilde{\zeta}^b = (\beta_s - \beta_b)$ is the value at which the borrowers value a marginal relaxation of the credit constraint.

[39] Although the savers labor-to-nondurable consumption ratio is not presented here, the intuition behind how it's determined is completely analogous to the borrowers' case.

[40] These are the same assumptions I impose when calibrating the model.

[41] Specifically, $\frac{\tilde{n}}{\tilde{c}^b} \geq 1 + \delta\frac{\tilde{d}^s}{\tilde{c}^s}$.

is mostly financed via labor income. As long as labor income is not too large (i.e. wages are sufficiently low so that labor income is mostly driven by labor supplied), borrowers cannot afford higher consumption levels relative to savers.

Given the previous discussion and given the parameter values α, χ, δ, β_s, β_b, ν_s and ν_b; expressions (2.3.21)-(2.3.27) can be used to solve for the vector of steady state variables $\tilde{S}$.[42]

2.4 Results

This section presents the chapter's main numerical results. I begin the section by discussing the model's calibration and then move on to discuss the results using the models's impulse response functions. The results are derived by solving the log-linearized system of equations (2.2.11), (2.3.3), (2.3.10), (2.3.11), (2.3.12), (2.3.13), (2.3.14), (2.3.15),(2.3.16), (2.3.17), (2.3.18), (2.3.19), (2.3.20), and the market clearing conditions for the non-durable good, durable good, nominal debt, and labor markets. The system is log-linearized around the deterministic steady state presented in Section 2.3.4 and its solution is computed following the method proposed by Sims (2002).

2.4.1 Calibration

In what follows, I use the term "baseline" to refer to the version of the model where no agent is borrowing constrained (i.e. all agents are savers); $\Omega_s \to 1$ and $\Omega_b \to 0$. This baseline version corresponds to the canonical New Keynesian model.

The model is specified at a quarterly frequency and it is calibrated so that certain variables of the deterministic steady state of the baseline version match specific targets. Table 2.1 summarizes the parameter values that result from the calibration of the baseline version of the model. The annual interest rate is targeted at 3%, which implies a time discount factor $\beta_s = 0.99$ for the savers. The annual depreciation rate of the durable good is set at 4%, which pins down the value of $\delta = 0.01$. Both types of agents are assumed to supply one third of their total time endowment; the disutility of labor parameters are then given by $\nu_s = 4.92$ and $\nu_b = 8.23$.[43] The other preference parameter, $\alpha = 0.69$, is calibrated so that the total share of private spending on the durable good is 20%.

[42] In calibrating the model, rather than specifying the value of the scaling parameters ν_s and ν_b, the scale of the economy is determined instead by fixing $\tilde{n}^b = \tilde{n}^s = 1/3$.

[43] The total time endowment for both agents is normalized to one; $H^b = H^s = 1$.

Table 2.1: Table of Parameter Values for the Baseline Model

Calibrated	
Parameter	**Value**
β_s	0.99
δ	0.01
ν_s	4.92
ν_b	8.23
ϵ_j	5.00
χ	0.29
α	0.69
Φ_c	0.75
Φ_d	0.50

Arbitrary	
Parameter	**Value**
β_b	0.98
θ	1.00
ϕ_π	1.50
ρ	0.50
κ	0.50

The parameter $\chi = 0.29$, which controls the fraction of the durable good that can be pledged as collateral, is determined by requiring a loan-to-value ratio of 70% for the borrowers.

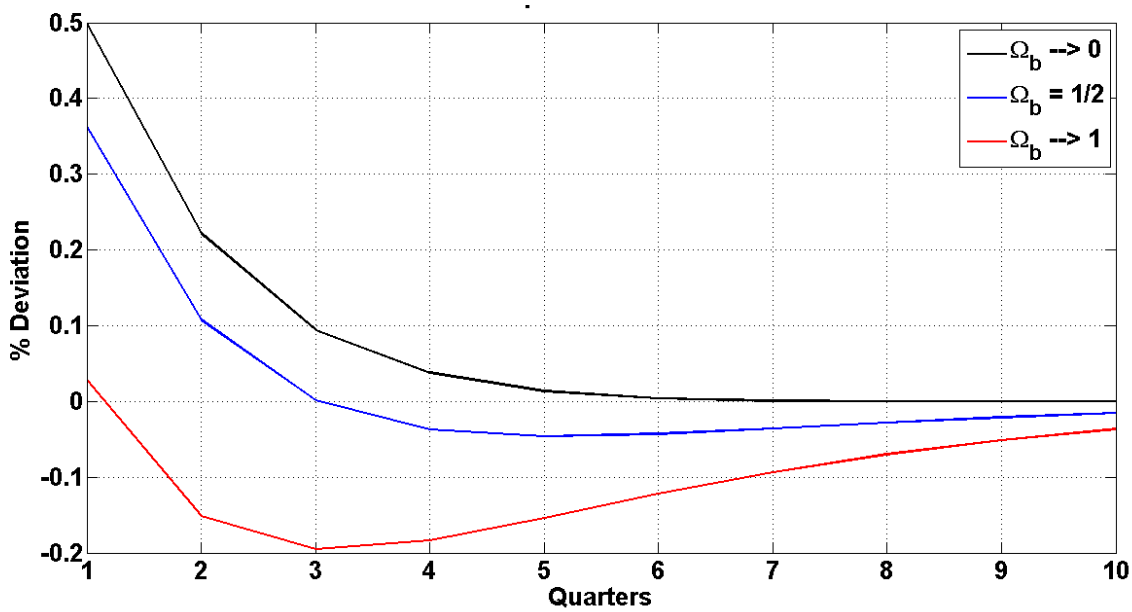
As for the parameters that enter the production side of the economy, Φ_j is chosen to match the average price adjustment frequencies in the durable and non-durable sectors. I assume that the durable sector can adjust prices more frequently; the durable sector adjustment frequency is set to two quarters ($\Phi_d = 0.5$) and the non-durable sector to four quarters ($\Phi_c = 0.75$). To simplify the analysis, both sectors have a price markup of 20%; which implies $\epsilon_c = \epsilon_d = 5$.

The rest of the agents' parameters are fixed at arbitrarily chosen values. Following Krusell and Smith (1998), the preference discount factor for the borrowers is set to $\beta_b = 0.98$. The inverse of the Frisch elasticity of labor supply is fixed at $\theta = 1$, which I impose so that the model more closely resembles the one presented on Chapter 1.

Regarding the parameters that control the behavior of the monetary authority, I assume that $\kappa = 0.5$; the durable and non-durable sectors are weighted equally in the composite inflation index. Following standard practice in the literature, I impose the Taylor principle and set $\phi_\pi = 1.5$. This assumption ensures the existence and uniqueness of a rational expectations equilibrium in a vast class of linear New Keynesian models.

Finally, the persistence of the monetary policy shock is set to $\rho = 0.5$. This is perhaps the only parameter value that is not within the range of standard values in the literature. In order to match some second moments of the data, the persistence of the monetary policy shock is usually much larger, $\rho \sim 0.95$. In this sense, the results presented on this chapter illustrate the effect of credit

Figure 2.2: Total Aggregate Output Response to a Negative Shock in the Taylor Rule

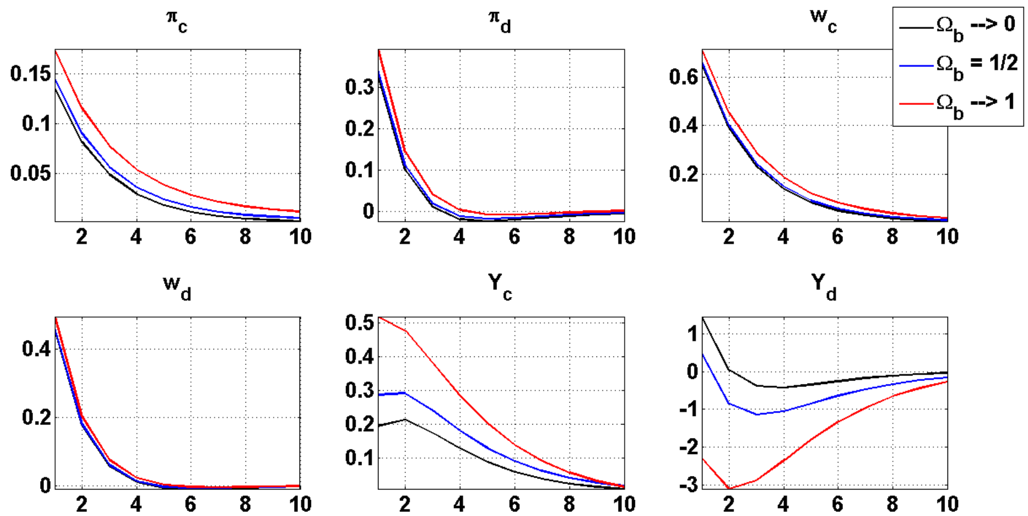


constraints under a mild transmission mechanism. Increasing the persistence of this exogenous process would further enhance the effect of credit constraints on the monetary policy transmission mechanism.

2.4.2 Impulse Response functions

The main result is given Figure 2.2, which illustrates the response of real total output to a 25 b.p. surprise decrease in the monetary policy rule. Total output measures overall production in the economy in units of the non-durable good; it is defined as $Y_t^T \equiv Y_{c,t} + q_t Y_{d,t}$. The baseline economy where no agent is borrowing constrained ($\Omega_b \to 0$) replicates the result of the canonical New Keynesian model; a negative monetary policy shock stimulates the economy by inducing a positive response in total output. This response is largest upon impact, but it remains positive for all subsequent periods. However, as the fraction of borrowing constrained agents increases ($\Omega_b = 0.5$), the stimulus to the economy becomes smaller. In the limiting case where all agents in the economy are credit constrained ($\Omega_b \to 1$), the negative monetary policy shock results in an economic contraction instead of an stimulus.

Figure 2.3: Price Response to a Negative Shock in the Taylor Rule

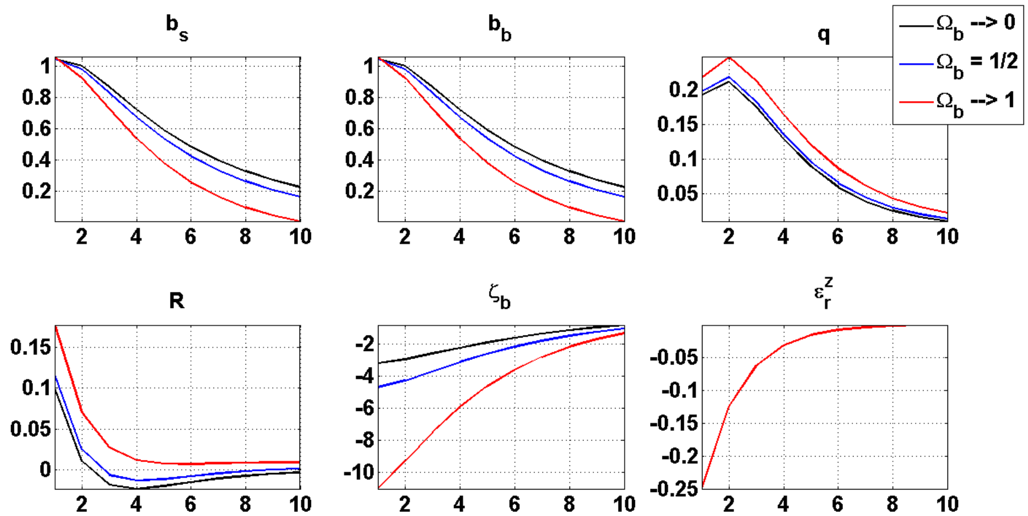


To understand the intuition behind this result, consider first the qualitative effect of the monetary policy shock on prices; as shown on Figure 2.3. Recall from Chapter 1 that the credit constraint has a rather small effect on equilibrium prices. Thus one can expect that, in the economy with credit constraints, prices react to the monetary policy shock in the same way they do in the canonical New Keynesian model. Sectorial inflation increases, but the increase is larger in the durables sector (the relatively more flexible sector). Thus, the relative price q increases as well. Note that larger prices are a consequence of higher productions costs; which is evinced in the increase of the wage rates w_c and w_d. Given that the nominal wage is the same in both sectors but the price level increases by more in the durables sector, the increase in the real wage rate in units of the durable good w_d is smaller than its counterpart in units of the non-durable good w_c.

Given the effect of the monetary policy shock on prices, agents adjust their bond holdings as shown in Figure 2.4. The adjustment is qualitatively similar in the canonical New Keynesian model ($\Omega_b \to 0$) and in the models with credit constraints ($\Omega_b = 0.5$ and $\Omega_b \to 1$). However, the force driving the adjustment is substantially different between the two models. In the canonical model, the increase in the borrowers' debt holdings is driven by the inter-temporal substitution effect.[44] In

[44]Strictly speaking, the canonical New Keynesian model is given by the case $\Omega_b = 0$. Under this scenario, the substitution effect is a consequence of the decrease in the nominal interest rate. The model with $\Omega_b \to 0$ is a very "close" approximation to the canonical model; the substitution effect also dominates in this model but the decrease in the nominal rate happens only from the second quarter onwards.

Figure 2.4: Bond Market Response to a Negative Shock in the Taylor Rule



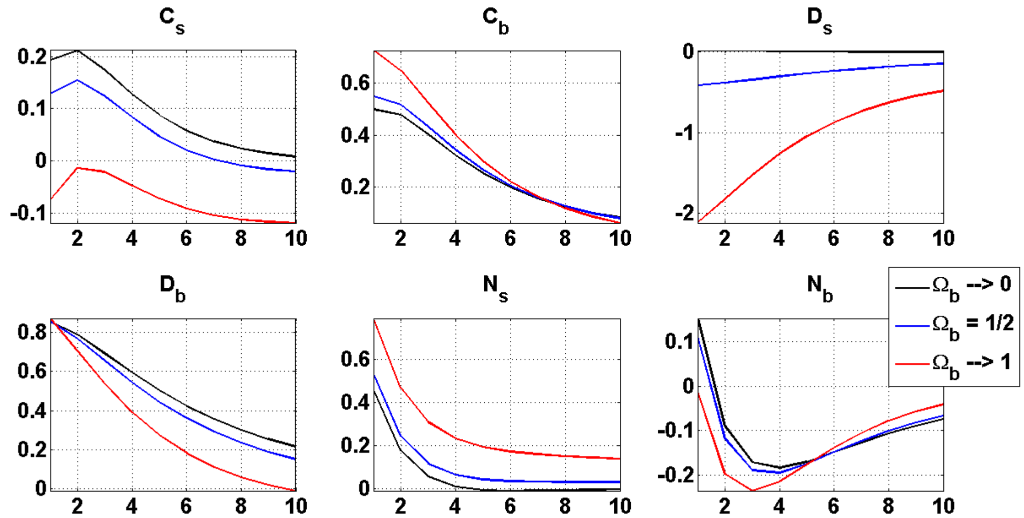
the model with borrowing constraints, the increase in the borrowers' debt holdings is driven by the increase in the collateral's market value; the larger relative price (q_t) and inflation $(\pi_{d,t})$ increase the durable good's market value. Thus the negative monetary policy shock relaxes the borrowing constraint, evinced by the decrease in the constraint multiplier ζ_t^b. As the constraint is relaxed, the borrowers are able to access more debt. Note that this leads to an initial increase in the nominal interest rate R_t; the debt market needs to clear and the increase in the nominal rate provides enough incentive for the savers to lend the additional funds to the borrowers.

The adjustment in the remainder of the borrowers' allocation is also qualitatively similar for the models with and without credit constraints; durable and non-durable consumption increase in response to the negative monetary policy shock as seen in Figure 2.5. However, in the presence of credit constraints, the responses are driven by wealth effects rather than substitution effects. As discussed in the previous paragraph, the borrowers' debt holdings increase due to the relaxation of the credit constraint. Larger debt effectively increases the borrowers' current wealth. In addition, the increase in the durable's relative price q_t implies a larger market value of the inherited durable holdings. The overall larger wealth is then responsible for the increase in durable and non-durable consumption. Note that the intuition behind this mechanism is exactly the same intuition described in Chapter 1; except that there I considered the case of going from no credit constraint to having a binding credit constraint (which is equivalent to a "tightening" of the credit constraint).

The savers' consumption allocation response is qualitatively different in the canonical New Keynesian model ($\Omega_b \to 0$) compared to the response in the models with credit constraints ($\Omega_b = 0.5$ and $\Omega_b \to 1$); as seen in Figure 2.5. As the fraction of credit constrained agents in the economy increases, the savers' non-durable and durable consumption responses become negative; the "expansionary" monetary policy shock can decrease overall consumption for the saver. As discussed in Chapter 1, this is a consequence of the indirect general equilibrium effects playing a more important role than the inter-temporal substitution interest rate effect. There are two competing indirect wealth effects. On one hand, there is a negative wealth effect driven by the additional lending to borrowers and the decrease in firm profits; the latter is mainly a consequence of higher production costs due to larger sectorial wage rates. On the other hand, the general equilibrium increase in the durable's relative price increases the market value of the inherited durable holdings. As the fraction of borrowing constrained agents increases, the negative wealth effect dominates; this explains why durable and non-durable consumption both decrease for large enough Ω_b. However, durable consumption responds negatively to the monetary "stimulus" even for small Ω_b; a consequence of additional negative substitution effects for the durable good. The increase in the durable's relative price makes non-durable consumption relatively more attractive. Furthermore, the increase in the nominal interest rate makes the nominal bond a relatively more attractive savings vehicle. Therefore, savers decrease their durable holdings since this good is now less attractive, both as a consumption good and as a wealth transferring device, even if they experience a positive wealth effect. Finally, note that when the fraction of credit constraint agents is very small ($\Omega_b \to 0$), the resulting positive wealth effect and the negative substitution effects cancel each other out and there is almost no deviation in the savers' durable consumption response.

To understand the aggregate effect of the monetary policy shock, consider first non-durable output Y_c. On one hand, the borrowers' non-durable consumption response to the monetary policy shock is similar to representative agent's consumption response in the canonical New Keynesian model regardless of the fraction of credit constraint agents; a negative monetary policy shock implies an increase in the borrower's consumption for all periods after impact. On the other hand, the savers' non-durable consumption response is similar to the representative agent's consumption response in the canonical New Keynesian model only when the fraction of credit constraint agents

Figure 2.5: Agents' Consumption Response to a Negative Shock in the Taylor Rule



is sufficiently small; a negative monetary policy shock leads to a contraction in the savers' non-durable consumption when Ω_b is too large. In the end, the non-durable output response is similar to the output response in the canonical New Keynesian model; the negative monetary policy shock leads to an stimulus in the non-durable sector, as shown by the panel "Y_c" in Figure 2.3. When the fraction of credit constraint agents is small (say the cases of $\Omega_b \to 0$ and $\Omega_b = 0.5$), both borrowers's and savers' effectively behave like the representative agent in the canonical New Keynesian model. When the fraction of credit constraint agents (borrowers) is large (say the case of $\Omega_b \to 1$), the borrowers drive the increase in non-durable output despite the savers' experiencing a contraction in non-durable consumption. Finally, note that although the the non-durable output response is similar to the output response in the canonical New Keynesian model, there is one feature that is different; the non-durable output response need not be the largest upon impact (which is evident from the cases $\Omega_b \to 0$ and $\Omega_b = 0.5$). This is is a consequence of the initial increase in the nominal interest rate; savers need this incentive to be willing to lend more funds to borrowers as their credit constraint relaxes due to the increase in the collateral value. The larger nominal rate upon impact has a slightly negative inter-temporal substitution effect on current non-durable consumption.

Consider next aggregate durable output Y_d, which is also given in Figure 2.3. Again, the borrowers' durable consumption response to the monetary policy shock is similar to representative agent's consumption response in the canonical New Keynesian model; regardless of the fraction of credit constrained agents. A negative monetary policy shock implies an increase in the borrower's durable consumption for all periods after impact. However, the savers' durable consumption response is completely different from the consumption response of the representative agent in the canonical New Keynesian model; as long as there are some credit constrained agents, a negative monetary policy shock implies a contraction in the savers' durable consumption for all periods. Recall that when the fraction of credit constraint agents is very small ($\Omega_b \to 0$), the interaction of the indirect wealth and substitution effects lead to no adjustment in the savers' durable consumption as these effect cancel each other out. As the fraction of credit constrained agents increases, the indirect general equilibrium effects imply a larger contraction in the savers' durable consumption. Overall, the decrease in savers' durable consumption is responsible for the contraction in the durable output sector, as seen in panel Y_d of Figure 2.3. Note that the contraction gets worse as the fraction of credit constrained agent increases; despite less savers in the economy, the decrease in their durable holdings is large enough to account for the total durable output decrease. Note that the durable output response is somewhat attenuated upon impact (for instance, for the cases $\Omega_b \to 0$ and $\Omega_b = 0.5$, the aggregate durable output response is positive upon impact). Again, this is a consequence of the initial increase in the nominal interest rate; there is a slightly positive inter-temporal substitution effect given the durable's storability property.

Finally, the effect on total output Y^T presented in Figure 2.2 is given by the combination of the effects on non-durable (Y_c) and durable (Y_c) output. When the fraction of the population that is credit constrained is relatively small (say the cases $\Omega_b \to 0$ and $\Omega_b = 0.5$), the monetary policy shock affects total output as it does in the canonical New Keynesian model. A negative monetary policy shock leads to an expansion in total output, which is driven mostly by the increase in production in the non-durable sector. In turn, this increase in non-durable output is mostly a consequence of the increase in borrowers' consumption that results from a relaxation in their credit constraint. However, when there are enough credit constrained agents (for instance the case $\Omega_b \to 1$), total output response differs substantially from the output response in the canonical New Keynesian model; a negative monetary policy shock leads to a *contraction* of total output. The contraction is mostly driven by the decrease in savers' durable consumption; a consequence of large negative wealth and substitution effects that arise from indirect general equilibrium adjustments.

Overall, even for the cases where the fraction of credit constrained agents is small, the monetary policy transmission mechanism is different than the mechanism in the canonical New Keynesian model. In the later, the consumption response is mostly driven by the inter-temporal substitution effect arising from the adjustments in the real (nominal) interest rate. In my model with credit constraints, the transmission mechanism depends on the agents' type. For borrowers, the consumption adjustments following the monetary policy shock are mostly a consequence of the changes in the collateral's market value; which depends not only on the real interest rate but also on the relative price of the durable good. For savers, the consumption adjustments are mostly driven by wealth effects arising from the adjustment in firms' profits (a consequence of wage rate adjustments) and substitution effects arising from changes in the real interest rate *and* the durable's relative price.

2.5 Conclusion

I investigate the effect of credit constraints on the monetary policy transmission mechanism using a standard two-sector New Keynesian model. The model features two types of agents who differ on their degree of patience, a durable and non-durable production sectors, nominal pricing frictions à la Calvo in each sector, a nominal bond, and a monetary authority which is introduced via a Taylor rule. The credit constraint is introduced as a collateral constraint that limits borrowing via the nominal bond; the relatively more impatient agents must use the durable good as collateral to back their borrowing.

Given this set up, I show that the monetary policy transmission mechanism in the model with credit constraints is significantly different from the transmission mechanism in the canonical New Keynesian model without financial frictions. In the later, the consumption response following a monetary policy shock is mostly driven by the inter-temporal substitution effect arising from the adjustments in the real (nominal) interest rate. In my model with credit constraints, the transmission mechanism depends on the agents' type. For the more impatient agents (who face the credit constraint), the consumption adjustments following the monetary policy shock are mostly a consequence of the changes in the collateral's market value; the relative price of the durable good adjusts in response to the monetary policy shock. For the more patient agents (who don't face the credit constraint), the consumption changes are driven by wealth and substitution effects arising from indirect general equilibrium adjustments. The wealth effects are mostly due to two factors. First, the change in agents' nominal asset position; the bond market clearing implies the patient agents must adjust their bond holdings to meet the borrowers' adjustment in debt. Second, the change in firms' profits; production costs change as real wages adjust to clear the labor market. Although there is an inter-temporal substitution effect that arises due to the change in the real (nominal) interest rate, this effect is secondary compared to the intra-temporal substitution effect arising from adjustments in the durable's relative price.

Furthermore, I show that this difference in the monetary policy transmission mechanism is relevant from an aggregate perspective; when the number of credit constrained agents is sufficiently large, a surprise negative monetary policy shock (traditionally associated with a monetary stimulus) can have contractionary effects. When the fraction of the population that is credit constrained is relatively small, the monetary policy shock affects total output as it does in the canonical New Keynesian model. The increase in total output resulting from the negative monetary policy shock is driven by the increase in non-durable output. In turn, the increase in non-durable output is mostly a consequence of the increase in the impatient agent's consumption that results from a relaxation in their credit constraint. However, when there fraction of the population that is credit constrained is sufficiently large, total output response differs substantially from the output response in the canonical New Keynesian model; a negative monetary policy shock leads to a *contraction* of total output. The contraction is mostly driven by the decrease in savers' durable consumption; a consequence of large negative wealth and substitution effects that arise from indirect general equilibrium adjustments.

Overall, the simple exercise conducted in this chapter illustrates that financial frictions (here in the form of credit constraints) can have very important implications for the monetary policy transmission mechanism and its effect on aggregate output.

CHAPTER 3

An Empirical Assessment of the Transmission of Monetary Policy

Abstract

In this chapter I conduct an empirical study of the transmission of monetary policy via credit costs. I consider three channels of transmission; the term structure of market interest rates (conventional channel), the spread over risk-free rates due to financial frictions (credit spread channel), and the non-spread credit conditions such as credit limits (non-spread credit channel). I am able to quantify the contribution of each channel by using an external instrument approach for the identification of the monetary policy surprise. I find that the credit channel can account for about 20% of the variance of durable expenditures and about 30% of the variance of non-durable expenditures following unanticipated monetary policy announcements. In addition, the non-spread factors can account for as much as half of the total contribution of the credit channel.

JEL Codes: E43, E44, E52, G10

3.1 Introduction

Over most of the post-war period, the Federal Reserve has conduced monetary policy by adjusting the Federal Funds rate. The rationale being that adjustments in the Federal Funds rate would in turn lead to adjustments in market interest rates and credit costs. Thus, with this indirect effect on credit/asset markets, the Federal Reserve could influence aggregate borrowing, saving, and consumption outcomes. The actual mechanism by which the changes (or announcements about changes) in the Federal Funds rate translate into changes in the credit costs is what I refer to as the monetary policy transmission mechanism.

The conventional view on the monetary policy transmission mechanism is captured in the canonical New Keynesian model without financial frictions. In this instance, the transmission takes place via the conventional channel; credit costs are entirely reflected in the expected future path of short-term rates (Fed Funds rate). However, in the early 90's, a strand of literature exemplified by papers such as Bernanke (1993) or Bernanke and Gertler (1995) started inquiring about the role of credit market frictions. A few years later, the seminal contributions of Kiyotaki and Moore (1997) and Ben S. Bernanke, Mark Gertler and Simon Gilchrist (1999) provided a micro-founded link between credit market frictions and the resulting credit costs. In both cases, the outcome is that credit costs incorporate a credit spread over the short-rate which depends on the net worth (asset position) of agents. In this context, monetary policy can be transmitted via the credit spread channel. To the extent that monetary policy can affect the net worth of agents, it can lead to changes in credit spreads. Much of the current literature still builds upon either of these two frameworks and, as a result, most models of policy transmission reflect only these two channels of transmission via credit costs; the conventional and credit spread channels.

On the empirical side, a large strand of literature has been devoted to assess the importance of each of these two channels for the transmission of monetary policy. Much of the recent work, such as Jiménez et al. (2012), Ciccarelli, Maddaloni and Peydró (2015), and Gertler and Karadi (2015), focuses on the question of whether the credit channel is quantitatively relevant for the transmission of monetary policy. In this chapter, I revisit this question and find that, as suggested by this literature, the credit channel's contribution for the transmission of monetary policy is as large, if not larger, than that of the conventional channel. In addition, I complement the existing work by exploring whether, within the credit channel, there is an additional transmission sub-channel (the credit non-spread channel) that is quantitatively relevant for the transmission of monetary policy. My motivation follows from the observation that, as pointed out earlier, most of the theory developed to explain the credit channel limits financial frictions to be solely reflected via credit spreads. However, a considerable portion of the tightening/relaxing of credit conditions takes place through adjustments in factors other than credit spreads. This is of particular importance for household credit, in which credit limits, credit scores or credit score exceptions seem to be the more relevant factors. I find that this non-spread sub-channel accounts for about half of the total contribution of the credit channel. This would suggest that including non-spread transmission channels in the modeling of monetary policy transmission is important. Furthermore, including such channels might be a step towards resolving the critique first proposed by Kocherlakota (2000) that financial frictions à la Kiyotaki and Moore (1997) or Ben S. Bernanke, Mark Gertler and Simon Gilchrist (1999), although qualitatively attractive, are quantitative unimportant.[45]

The approach I take combines the strengths of two strands of empirical literature. First, as in Ciccarelli, Maddaloni and Peydró (2015) and Jiménez et al. (2012), I use survey data obtained from banks in order to measure the overall credit conditions. In this sense, I allow for a more general scope of credit costs rather than restricting them to be solely captured by credit spreads. Second, as in Gertler and Karadi (2015), I use an external instrument identification approach for the monetary policy shock rather than the recursive identification scheme à la Christiano, Eichenbaum and Evans (1996). This allows me to avoid making strong a priori restrictions about the nature of the interaction between the policy instrument and the rest of the variables in the model.

I study the monetary policy transmission mechanism in the context of a reduced form vector autoregression (VAR) that includes three type of variables: nominal, real and financial. Within the financial factors, I use variables constructed using data from the Senior Loan Officer Opinion Survey (SLOOS) as well as credit spreads associated with three important credit markets: commercial and industrial loans, mortgage loans, and commercial paper loans. I proceed in two steps. First, I show that the SLOOS variables contain additional information about the credit conditions, which is not reflected in the credit spreads. I attribute this additional information to the non-spread factors of

[45]The financial accelerator and collateral constraint type of frictions yield purely spread-based transmission mechanisms.

credit costs; credit score and collateral requirements, credit limits, etc. Second, taking advantage of the additional information in the SLOOS variables, I quantify the contribution of the credit channel and each of its two sub-channels (credit spread and non-spread) for monetary policy transmission.

The first result about the additional information of the SLOOS variables is reflected in two aspects. First, following an unanticipated monetary policy announcement, the credit spreads capture mostly short-term changes in the credit conditions. However, the SLOOS variables reflect short-term and long-term changes. Second, the credit spreads can only account for a "common" component of credit conditions in the commercial and industrial, mortgage, and household credit markets (with a slight bias toward the commercial and industrial loans market). There are other credit conditions, specific to the mortgage and household credit markets, that are captured by the SLOOS variables which are not reflected in the credit spreads. Importantly, an unanticipated monetary policy announcement does significantly affect these other credit conditions.

My second result is that the changes in credit conditions that follow an unanticipated monetary policy announcement play a significant role for the monetary policy transmission. This credit channel is particularly important for the reaction of non-durable and durable expenditures. To put it in perspective, the contribution of the credit channel can be as large as the contribution of the conventional channel, accounting for about 20% of the transmission for durable expenditures and 30% for non-durables. Additionally, a misspecified VAR model that doesn't include any financial sector does not only fail to capture the credit channel, but it over-emphasizes the importance of the conventional channel. This last observation is most relevant for non-durable expenditures, where the contribution of the conventional channel can be overstated by as much as 20%.

Finally, my third result suggests that the composition of the credit channel is relevant. I find that within the credit channel, roughly half of the monetary policy transmission can be attributed to changes in credit spreads and half to changes in other credit conditions (although for non-durables the contribution of the non-spread component can be as large as two thirds). Interestingly, most of the contribution of the non-spread component is captured by the SLOOS credit demand variables. However, when only the credit spreads are used as proxies of the financial sector, the effect of the non-spread component is attributed instead to changes in the commercial paper spread. This finding suggests that there might be an omitted common factor which affects both, commercial paper spreads and credit demand for mortgages and household credit.

Altogether, these results are further evidence of the importance of financial frictions in the modeling of monetary policy transmission. The novel contribution of this chapter relates to the composition of the credit channel. Models that restrict financial frictions to be solely embodied by credit spreads might fail to capture a large fraction of the transmission of unanticipated monetary policy news to aggregate activity; specially for non-durable and durable good expenditures. Models which explicitly incorporate the non-spread channels (credit limits, credit scores, down payments) might have a better change of fitting the data.

The rest of the paper is organized as follows. Section 3.2 introduces the data for the empirical analysis. I provide a detailed description of the relationship between the SLOOS and the credit spreads. Section 3.3 formally introduces the econometric framework I use. I discuss the external instrument approach used for the identification of the monetary policy shock along with the modified Forecast Error Variance Decomposition. I show how these tools allow me to quantify the contribution of the different channels for monetary policy transmission. Section 3.4 discusses the different VAR specifications I estimate along with my choice of instruments. Section 3.5 discusses the main results of the paper. Finally, Section 3.6 presents my concluding remarks.

3.2 Data

I analyze monthly data that includes economic, financial, and credit variables. The data spans the period 1990:1 to 2012:6.[46] The economic variables include measures of aggregate economic activity and prices. In particular, I use data on industrial production (IP), durable (D) and non-durable (C) expenditures, households' debt (B), and the CPI as the measure of aggregate prices. When included in the VAR specification, all these variables are logged. In addition the aggregate activity variables are deflated using the CPI.[47]

For the monetary policy indicator, I use the one-year rate.[48] As it will be explained in more detail in Section 3.3, there is a difference between the policy instrument and the policy indicator. The use of the one-year rate as the policy indicator does not imply that the Federal Reserve conducts policy

[46]The sample period is selected purely for reasons of data availability. The survey data used to construct the credit variables is only available starting in 1990:1. In addition, some of the credit spreads and the external instruments used for the high frequency identification are taken from previous studies, such as Gilchrist and Zakrajšek (2012), Gürkaynak, Sack and Swanson (2005), and Gertler and Karadi (2015); and thus are available only through 2012:6.

[47]The IP and CPI variables were taken from FRED while C, D, and B where obtained from FRB Flow of Funds.

[48]The discussion presented in the paper is based on the one-year rate as the policy indicator. However, robustness checks were done using the short-term and two-year rates and the results continue to hold.

by directly manipulating this rate. As the general consensus dictates, I presume that the Federal Reserve conducts policy by directly controlling the federal funds rate (i.e. the policy instrument). However, through the usual term structure argument, any movements in the federal funds rate affect the one-year rate. In this sense, the one-year rate is an indicator of the monetary policy stance. The advantage of using this mid-term rate is that it, in addition to reflecting current movements in the policy instrument, it captures movements in its expected future *path*.

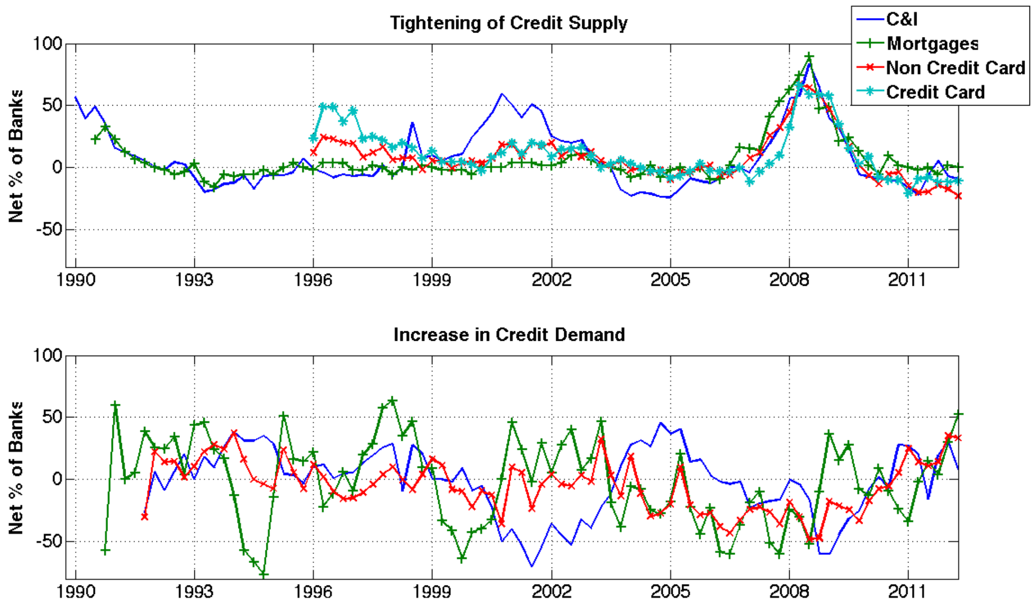
For the financial sector, I use three different credit spreads: the excess bond premium (EBP) constructed by Gilchrist and Zakrajšek (2012), a mortgage spread (SP_{Mort}) and the three-month commercial paper spread (SP_{CP}). These variables pertain to three important financial markets in the U.S. The EBP reflects the long-term credit cost for the non-farm production sector. The SP_{Mort} is a proxy for the cost of home ownership. Finally, the SP_{CP} is relevant for the cost of short-term business credit and consumer durable purchases.[49]

For the credit sector, I use data from the Senior Loan Officer Opinion Survey (SLOOS) to construct seven variables that reflect the credit demand and supply conditions for the economy. I construct credit supply variables for four different loan types: commercial and industrial ($C\&I_S$), mortgages ($Mort_S$), households' non-credit card (NCC_S), and households' credit cards (CC_S). Similarly, I construct credit demand variables for three different loan types: commercial and industrial ($C\&I_D$), mortgages ($Mort_D$), and households' loans (HH_D, which include both, credit and non-credit card).[50] The credit supply variables are constructed as the *net percent* of banks that "tightened" the standards for approving the different loan types during the quarter. A positive/negative value indicates that it is relatively hard/easy to get this particular type of loan because, overall, banks now require tougher/weaker standards. A larger magnitude is associated with a larger number of banks that require tougher/weaker standards. The credit demand variables are constructed in a similar way. For each of the three different loan types, they measure the *net percent* of banks that reported a "stronger" demand for loans. A positive/negative value is associated with an increase/decrease in aggregate demand for loans. The magnitude of the variable reflects the number of banks that reported an increase/decrease in the demand for the different loans.

[49]The mortgage and the three-month commercial paper spreads are taken from Gertler and Karadi (2015).

[50]Although the SLOOS contains quarterly data, I convert it to a monthly frequency. For the discussion presented here, the monthly time series is constructed by assuming that for the months within a quarter, the values are held fixed at their quarterly level. The results are robust to alternative interpolation procedures (i.e. linear or cubic splines).

Figure 3.1: Credit Demand and Supply Measures from the SLOOS (*Quarterly, 1990:1-2012:2*)



Note: SLOOS data about credit supply conditions for households' credit and non-credit card loans is available only from 1995:4. Data about credit demand conditions for C&I and households's credit and non-credit card is available only from 1991:3.

Figure 3.1 displays the seven SLOOS variables. As seen in the top panel, some of the credit supply tightening episodes coincide with recessionary periods: the savings and loans crisis of 1989 and the early 90's recession, the global stock mini-crash of 1997 in the wake of the Asian Crisis, the early 2000's recession, and the 2008 financial crisis. There are some additional features of the SLOOS data that are worth pointing out. First, the credit demand variables exhibit a much larger volatility than their supply counterparts. Second, the credit supply variables are highly correlated with each other. The correlation coefficients range between 0.479 and 0.879. The highest correlations occur between the variables that capture the credit standards for household loans: mortgages, credit card and non-credit card (with correlation coefficients in excess of 0.75). Note that the demand side variables seem to be correlated as well, but not as strongly. The correlation coefficients for this block are only in the range of 0.354 to 0.449. Finally, there seems to be some negative correlation between the credit demand and supply variables.[51]

The large overall correlation displayed by the credit variables has some important consequences for the way in which I perform the empirical analysis. In order to avoid multicollinearity when estimating the different VAR specifications, I use a principal component decomposition as an or-

[51]Table 5.1 in Section **??** presents the corresponding correlation coefficients.

thogonalization procedure for the credit variables. For each of the two blocks, credit supply and demand, I extract the principal components and use those as the measures of credit supply and demand conditions.[52] The principal components for the credit supply block are given by PC_S^1, PC_S^2, PC_S^3, and PC_S^4. The first principal component $\left(\text{PC}_\text{S}^1\right)$ reflects the overall credit supply conditions for all loan types, with a slight bias for the commercial and industrial loans. The second principal component $\left(\text{PC}_\text{S}^2\right)$ captures mostly the credit supply conditions for the commercial and industrial loans and credit card loans. The third $\left(\text{PC}_\text{S}^3\right)$ and fourth $\left(\text{PC}_\text{S}^4\right)$ principal components reflect the credit supply conditions for the mortgage and non-credit card loans, respectively. For the credit demand block the principal components are PC_D^1, PC_D^2, and PC_D^3. Unlike the case of the supply block, there is no common component and each principal component is mostly associated with a specific loan type. The first $\left(\text{PC}_\text{D}^1\right)$ and third $\left(\text{PC}_\text{D}^3\right)$ principal components reflect the conditions for households' loans (mortgages, credit card and non-credit card loans); while the second principal component $\left(\text{PC}_\text{D}^2\right)$ is mostly associated with commercial and industrial loans.[53]

Given the survey nature of the SLOOS, there are two questions that naturally arise. First, whether or not the SLOOS data contains relevant information about the credit conditions in the economy. Second, if the SLOOS data is indeed relevant, the extent to which this information is different from the information captured by other measures of credit costs, such as credit spreads. In order to shed some light into the issue, Tables 3.1 and 3.2 present some results about the explanatory power of the SLOOS variables on the credit spreads and vice-versa.

The results about the explanatory power of the SLOOS variables on the credit spreads are summarized in Table 3.1. The table presents the output of simple regressions of each of the credit spreads on all the non-orthogonalized SLOOS variables.[54] These results suggest a very large explanatory power of the SLOOS variables for the EBP and mortgage spreads. The EBP is mostly explained by the credit supply conditions for all four different loan types and by the credit demand conditions for the commercial and industrial loans (all these variables have coefficients that are significant at the 1 percent level). As for the mortgage spreads, the relevant information is captured

[52]The results are robust to alternative orthogonalization procedures, such as using the residuals of simple regressions within each block.

[53]Table 5.2 in Section **??** presents the loading factors for the principal component decomposition.

[54]The relatively large explanatory power of the SLOOS variables on the credit spreads is robust to the orthogonalization procedure. Depending on how the data is orthogonalized, the Adj. R^2 ranges are given by: 0.637-0.680 for the EBP, 0.472-0.634 for the SP_Mort, and 0.361-0.381 for the SP_CP.

Table 3.1: Explanatory Power of SLOOS on Credit Spreads (*Monthly, 1995:10-2012:6*)

	EBP	SP_{Mort}	SP_{CP}
$C\&I_S$	0.015^{***}	0.010^{***}	0.006^{***}
	(7.383)	(10.077)	(6.320)
$Mort_S$	0.008^{***}	0.006^{***}	-0.001
	(3.230)	(4.873)	(-0.766)
NCC_S	-0.013^{***}	-0.006^{*}	0.004
	(-2.855)	(-1.961)	(1.426)
CC_S	0.010^{***}	-0.001	-0.001
	(3.017)	(-0.808)	(-0.402)
$C\&I_D$	-0.008^{***}	-0.001^{*}	0.005^{***}
	(-5.002)	(-1.802)	(6.552)
$Mort_D$	0.001	-0.001^{**}	-0.001^{**}
	(0.649)	(-2.519)	(-2.322)
HH_D	0.002	0.003^{***}	-0.002^{*}
	(1.091)	(3.332)	(-1.794)
Observations	201	201	201
Adj. R^2	0.680	0.634	0.369
F-statistic	55.435	63.434	17.162

Note: Robust t-statistics in parenthesis
*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

by the credit supply conditions for commercial and industrial and mortgage loans (with coefficients that are significant at the 1 percent level) and the credit demand conditions for all types of household loans (with coefficients that are significant at the 5 percent level). Finally, for the commercial paper spread, the SLOOS variables have some explanatory power but not as much as for the other two spreads (with an Adj. R^2 of only 0.369). Interestingly, the credit demand conditions seem to be the relevant ones in explaining this spread (all three credit demand variables have coefficients that are significant at the 10 percent level). Overall, the information contained in the credit spreads seems to be explained relatively well by the SLOOS variables.

Table 3.2 summarizes the results about the explanatory power of the credit spreads on the SLOOS variables. The output presented corresponds to simple regressions of each of the non-orthogonalized SLOOS variables on the three credit spreads. These results suggest a limited explanatory power of the credit spreads. The only variable they adequately explain is the credit supply for commercial and industrial loans, with an Adj. R^2 of 0.638. For the other credit supply and

Table 3.2: Explanatory Power of Credit Spreads on SLOOS variables (*Monthly, 1990:1-2012:6*)

	$C\&I_S$	$Mort_S$	NCC_S	CC_S	$C\&I_D$	$Mort_D$	HH_D
EBP	20.063^{***}	8.872^{***}	15.181^{***}	19.062^{***}	-24.974^{***}	18.381^{***}	-2.231
	(7.696)	(5.044)	(8.311)	(9.639)	(-8.950)	(5.052)	(-0.863)
SP_{Mort}	18.861^{***}	13.279^{***}	-4.365	-20.527^{***}	-15.633^{***}	-20.970^{**}	-6.799
	(4.344)	(2.972)	(-0.762)	(-3.212)	(-2.659)	(-2.207)	(-1.196)
SP_{CP}	22.933^{***}	21.544^{***}	21.220^{***}	18.155^{***}	17.421^{***}	-44.123^{***}	-25.798^{***}
	(5.082)	(4.937)	(5.881)	(4.534)	(3.327)	(-5.489)	(-6.438)
Observations	270	267	201	201	252	264	252
Adj. R^2	0.638	0.406	0.443	0.333	0.477	0.137	0.147
F-statistic	73.591	26.897	60.332	48.076	96.675	19.464	22.273

Note: Robust t-statistics in parenthesis. Number of observations constrained by SLOOS data availability.
*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

demand variables, the explanatory power of the credit spreads is only modest, with the Adj. R^2 ranging between 0.137 to 0.477. Interestingly, the explanatory power of the credit spreads is the smallest for the variables that pertain to the credit demand conditions for household loans (mortgages, non-credit card and credit card loans).[55] In conclusion, a large amount of the information contained by the SLOOS variables can't be explained by the credit spreads.

From a heuristic standpoint, the results presented in Tables 3.1 and 3.2 provide evidence about the larger information content of the SLOOS variables. This additional information might stem from other non-spread factors that affect credit conditions, such as credit limits, collateral requirements, credit score requirements, credit score exceptions, just to name a few.

3.3 Econometric Framework

The model I use is a vector autoregression (VAR) which includes nominal, real, and credit variables. For the identification of the monetary policy shock, I use the external instrument methodology proposed by Gertler and Karadi (2015). This approach relies on the ideas of high frequency identification (HFI) applied within the context of a VAR.[56] I use this procedure for three reasons. First, it allows me to include financial and credit variables in the VAR specification without imposing a

[55] When the regressions from Table 3.2 are done using the orthogonalized SLOOS variables, the credit spreads are only able to explain one of the components of each of the credit supply and demand blocks. For both blocks, the Adj. R^2 of the "main" component regression on the credit spreads is of about 0.6. The remaining components are very poorly explained by the credit spreads, with an Adj. R^2 ranging from 0.01 to about 0.06.

[56] Some papers that make use of the HFI procedure in other contexts are Kuttner (2001), Gürkaynak, Sack and Swanson (2005), and Hamilton (2009).

priori restrictions on the interaction between them and the federal funds rate (or the policy instrument).[57] Second, depending on the choice of policy indicator and external instrument, the identified policy surprise can include shocks to forward guidance. That is, the identified surprise is informative not only about the current policy stance, but also about the expected future policy stance. Finally, in conjunction with an adaptation of the forecast error variance decomposition (FEVD), it allows me to quantify the contribution of each of the different channels for the monetary policy transmission.

Let $\mathbf{Z}_t$ be the vector of endogenous nominal, real, and credit variables; $\mathbf{A}, \mathbf{K}$ conformable coefficient matrices; $\mathbf{B}(L)$ a lag polynomial (of order p) conformable matrix; and $\boldsymbol{\epsilon}_t$ a vector of structural white noise innovations with identity covariance matrix. The structural VAR that I consider is given by

$$\mathbf{A}\mathbf{Z}_t = \mathbf{K} + \mathbf{B}(L)\,\mathbf{Z}_{t-1} + \boldsymbol{\epsilon}_t. \tag{3.3.1}$$

Assuming that the matrix $\mathbf{A}$ is invertible so that $\mathbf{S} = \mathbf{A}^{-1}$, the reduced form representation of (3.3.1) is then

$$\mathbf{Z}_t = \tilde{\mathbf{K}} + \tilde{\mathbf{B}}(L)\,\mathbf{Z}_{t-1} + \mathbf{u}_t, \tag{3.3.2}$$

with $\tilde{\mathbf{K}} \equiv \mathbf{S}\mathbf{K}$, $\tilde{\mathbf{B}}(L) \equiv \mathbf{S}\mathbf{B}(L)$, and the vector of reduced form shocks $\mathbf{u}_t = \mathbf{S}\boldsymbol{\epsilon}_t$. Note that the covariance matrix of the reduced form shocks is given by

$$\boldsymbol{\Sigma} = \mathbb{E}_t\left[\mathbf{u}_t\mathbf{u}_t'\right] = \mathbb{E}_t\left[\mathbf{S}\mathbf{S}'\right]. \tag{3.3.3}$$

Define the monetary policy indicator $z_t^p \in \mathbf{Z}_t$ as the variable in the structural representation (3.3.1) associated with the fundamental policy shock $\epsilon_t^p \in \boldsymbol{\epsilon}_t$.[58] Note that one can write $\boldsymbol{\epsilon}_t = [\epsilon_t^p \;\; \tilde{\boldsymbol{\epsilon}}_t]'$

[57] Note that the identification scheme of Christiano, Eichenbaum and Evans (1996) assumes that within a period the fed funds rate responds to all the variables in the VAR but not vice-versa. For aggregate activity variables, such as prices or real output measures, this assumption might be justified if the frequency of the data is not too low (monthly or quarterly). However, for financial and credit variables, this assumption is less likely to hold; even for monthly or quarterly data.

[58] As Gertler and Karadi (2015) note, there is a distinction between the monetary policy indicator and the monetary policy instrument. The latter is the current period federal funds rate, through which the monetary authority conducts its policy. The former is a proxy of the stance of monetary policy. By using a policy indicator with a longer maturity than the current period fed funds rate (such as the yield on one year government bonds) one can capture shocks to forward guidance. This is because longer maturity indicators reflect not only shifts in the current fed funds rate but also shifts in expectations about the future path of short rates.

and $\mathbf{S} = \begin{bmatrix}\mathbf{s}^p & \tilde{\mathbf{S}}\end{bmatrix}$, so that $\mathbf{u}_t = \mathbf{s}^p \epsilon_t^p + \tilde{\mathbf{S}}\tilde{\boldsymbol{\epsilon}}_t$. The column vector $\mathbf{s}^p$ captures the impact of the fundamental policy disturbance in each of the reduced form errors. Therefore, I interpret $s_j^p \in \mathbf{s}^p$ as an indicator of the contemporaneous propagation of the monetary policy shock via variable $z_j \in \mathbf{Z}$. This interpretation will allow me to quantify the relative contribution of the different channels for the propagation of unanticipated monetary policy news, as I show later.

Since I am interested in analyzing the response of the system in (3.3.1) to the fundamental monetary policy disturbance only, it suffices to identify and estimate the elements in $\mathbf{s}^p$ rather than the entire matrix $\mathbf{S}$. To this end, denote $\mathbf{s}^p = [s_p^p \ \tilde{\mathbf{s}}^p]'$, where s_p^p captures the contemporaneous propagation of the monetary policy shock via the monetary policy indicator. Consider IV_t, an external instrument satisfying the following two conditions:

$$\mathbb{E}\left[\text{IV}_t\, \epsilon_t^p\right] = \gamma \neq 0 \tag{3.3.4}$$

$$\mathbb{E}\left[\text{IV}_t\, \tilde{\boldsymbol{\epsilon}}_t\right] = \mathbf{0}. \tag{3.3.5}$$

That is, the external instrument must be correlated with the fundamental monetary policy disturbance (equation (3.3.4), the relevance condition) while being orthogonal to all the other structural disturbances (equation (3.3.5), the exogeneity condition).

Given the instrument IV_t, I proceed as follows. First, I estimate the reduced form VAR in (3.3.2) using OLS to obtain estimates of the parameter matrices $\widehat{\tilde{\mathbf{K}}}$ and $\widehat{\tilde{\mathbf{B}}}(L)$, the reduced form residual vector, $\hat{\mathbf{u}}_t = \begin{bmatrix}\hat{u}_t^p & \widehat{\tilde{\boldsymbol{u}}}_t\end{bmatrix}'$ (where $\hat{u}_t^p$ is the reduced form residual corresponding to the equation of the policy indicator), and its corresponding covariance matrix, $\widehat{\boldsymbol{\Sigma}}$. Second, I perform a two-stage least squares regression of $\widehat{\tilde{\boldsymbol{u}}}_t$ on $\hat{u}_t^p$ using the instrument IV_t:

$$\text{First Stage:} \qquad \hat{u}_t^p = \beta\, \text{IV}_t + \nu_t^p \tag{3.3.6}$$

$$\text{Second Stage:} \qquad \widehat{\tilde{\boldsymbol{u}}}_t = \boldsymbol{\alpha}\left(\hat{\beta}\, \text{IV}_t\right) + \tilde{\boldsymbol{\nu}}_t. \tag{3.3.7}$$

Intuitively, given assumption (3.3.4), the fitted value of the first stage regression isolates the variation of $\hat{u}_t^p$ that is due to the fundamental policy shock ϵ_t^p. Meanwhile, the second stage regression captures the variation of the remaining residuals that is due to ϵ_t^p. The advantage of using this two-stage approach is that it identifies the vector of contemporaneous propagation coefficients, $\mathbf{s}^p$, up to a scaling factor. This is because the coefficient of the second stage regression is given by

$\boldsymbol{\alpha} = \tilde{\mathbf{s}}^p \left(s_p^p\right)^{-1}$ and its OLS estimator, $\widehat{\boldsymbol{\alpha}}$, is consistent and unbiased whenever assumption (3.3.5) holds. The procedure is completed by pinning down the scaling factor s_p^p. As Gertler and Karadi (2015) show, the restriction on the covariance matrix, equation (3.3.3), implies that s_p^p can be identified up to a sign convention.[59]

Using the estimates $\hat{s}_p^p$, $\widehat{\boldsymbol{\alpha}}$, $\widehat{\tilde{\mathbf{K}}}$, and $\widehat{\tilde{\mathbf{B}}}(L)$ along with the reduced from representation (3.3.2), one can compute the impulse response functions (IRF's) of the system. Additionally, as I show next, one can use a procedure along the lines of a Forecast Error Variance Decomposition (FEVD) in order to quantify the contribution of the different channels for the monetary policy transmission mechanism.

Since I am only interested in studying the transmission of the fundamental monetary policy disturbance, in what follows, I assume $\tilde{\boldsymbol{\epsilon}}_t = \mathbf{0}$, $\forall t$. For any horizon h and using the moving average (MA) representation of (3.3.2), the forecast error for the vector $\mathbf{Z}_t$ can be written as

$$\mathbf{Z}_{t+h} - \mathbb{E}_t\left[\mathbf{Z}_{t+h}\right] = \sum_{q=0}^{h-1} \boldsymbol{\Psi}_q \mathbf{u}_{t+h-q} = \sum_{q=0}^{h-1} \boldsymbol{\Psi}_q \mathbf{s}^p \epsilon_{t+h-q}^p \tag{3.3.8}$$

with the MA coefficients defined recursively as $\boldsymbol{\Psi}_0 = \mathbf{I}$ and $\boldsymbol{\Psi}_q = \sum_{j=1}^{q} \boldsymbol{\Psi}_{q-j}\mathbf{B}_j, \forall q \geq 1$. Note that for $j > p$, $\mathbf{B}_j = 0$; while for $1 \leq j \leq p$, B_j denotes the j^{th} autoregressive coefficients in the reduced form representation $\left(\text{i.e } \tilde{\mathbf{B}}(L)\, \mathbf{Z}_{t-1} = \sum_{j=1}^{p} \mathbf{B}_j \mathbf{Z}_{t-j}\right)$.

Let n denote the number of variables in the vector $\mathbf{Z}$ and consider the variable $z_t^i \in \mathbf{Z}_t$. Denote by s_k^p the k^{th} component of the vector $\mathbf{s}^p$ and by $\psi_q^{i,k}$ the $\left(i^{\text{th}}, k^{\text{th}}\right)$ element of the matrix $\boldsymbol{\Psi}_q$. In light of (3.3.8), the forecast error variance of variable z_{t+h}^i due exclusively to the fundamental monetary policy shock, ϵ_t^p, is given by

$$\text{FEV}_i\left(h\right) \equiv \mathbb{V}ar\left(z_{t+h}^i - \mathbb{E}_t\left[z_{t+h}^i\right]\right) = \sum_{q=0}^{h-1}\left(\sum_{k=1}^{n} \psi_q^{i,k} s_k^p\right)^2. \tag{3.3.9}$$

Consider the monetary policy transmission upon impact (i.e. $h = 1$). Since $\psi_0^{i,k} = 1$ iff $i = k$ and zero otherwise, the one-step ahead forecast error variance is $\text{FEV}_i\left(1\right) = \left(s_i^p\right)^2$. The contemporaneous variation of variable z_t^i due to an unexpected monetary policy announcement is completely

[59]Furthermore, Gertler and Karadi (2015) provide a closed form solution that yields $\hat{s}_p^p$ as a function of the estimated parameters $\widehat{\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{\Sigma}}$.

summarized by the parameter s_i^p. Furthermore, the ratio

$$\phi_i = \frac{\text{FEV}_i\,(1)}{\sum_{k=1}^{n} \text{FEV}_k\,(1)} \tag{3.3.10}$$

measures the size the contemporaneous variation in variable z_i relative to the total variation induced by the unexpected monetary policy shock. In this sense, ϕ_i quantifies the contribution of variable z_i to the monetary policy transmission mechanism.

In order to extend the previous idea to a forecast horizon $h > 1$, define

$$\text{FEV}_{i,k}\,(h) \equiv \sum_{q=0}^{h-1} \left(\psi_q^{i,k} s_k^p\right)^2 \tag{3.3.11}$$

$$\text{FEV}_i^{\text{var}}\,(h) \equiv \sum_{k=1}^{n} \text{FEV}_{i,k}\,(h) \tag{3.3.12}$$

$$\text{FEV}_i^{\text{cov}}\,(h) \equiv \sum_{q=0}^{h-1} \left[\sum_{k=1}^{n} \left(\psi_q^{i,k} s_k^p\right) \sum_{r \neq k}^{n} \left(\psi_q^{i,r} s_r^p\right) \right]. \tag{3.3.13}$$

With $j \geq 1$, an unanticipated monetary policy shock $t + j$ periods ahead (ϵ_{t+j}) results in a contemporaneous change in variable z_{t+j}^k. Consider in addition variable $z^i \in \mathbf{Z}$. For $h > j$, a change in variable z^k *at time t+j* leads to variation in z^i *at time t+h*. Therefore, $\text{FEV}_{i,k}\,(h)$ quantifies the variation in z_{t+h}^i due to changes induced in variable z^k by contemporaneous monetary policy shocks from periods $t + j$ to $t + h$. In this sense, for any variable z^i, $\text{FEV}_{i,k}\,(h)$ quantifies the transmission of the monetary policy disturbance *via* variable z^k *alone*. It follows that $\text{FEV}_i^{\text{var}}\,(h)$ quantifies the variation in z_{t+h}^i due to *individual* changes induced in *all* the variables of $\mathbf{Z}$ by contemporaneous monetary policy disturbances from periods $t + j$ to $t + h$.

To understand the term $\text{FEV}_i^{\text{cov}}\,(h)$, consider an additional variable $z^r \in \mathbf{Z}$. Given the nature of the VAR, the contemporaneous change in variable z_{t+j}^k due to ϵ_{t+j} results in a *simultaneous* change in variable z_{t+j}^r as well. Therefore, for $h > j \geq 1$, there is some additional variation in z_{t+h}^i due to this simultaneous *joint* effect of the policy disturbance in variables z^k and z^r. Note that, within the current set up, there is no way of telling the direction of causality of the *simultaneous joint* change. That is, whether the induced change in z_{t+j}^k leads to the change in z_{t+j}^r or vice-versa. Therefore, $\text{FEV}_i^{\text{cov}}\,(h)$ quantifies the total transmission of the monetary policy disturbance *via* this covariance effect.

Using the previous definitions, the forecast error variance of variable z^i_{t+h}, equation (3.3.9), can be rewritten as $\text{FEV}_i(h) = \text{FEV}^{\text{var}}_i(h) + \text{FEV}^{\text{cov}}_i(h)$. The first term refers to the transmission of policy via induced *contemporaneous variance* on individual variables. It captures the transmission of monetary policy that can be attributed to a specific variable in $\mathbf{Z}$. The second term refers to the transmission of policy via induced *contemporaneous covariance* between variables. It captures the transmission of monetary policy that can't be associated with a specific variable within $\mathbf{Z}$.[60] In light of these observations, for variable z^i at forecast horizon h, I define

$$\gamma_i(h) \equiv \frac{\left|\text{FEV}^{\text{var}}_i(h)\right|}{\left|\text{FEV}^{\text{var}}_i(h)\right| + \left|\text{FEV}^{\text{cov}}_i(h)\right|} \tag{3.3.14}$$

$$\bar{\gamma}_i \equiv \frac{\sum_{j=0}^{h} \gamma_i(j)}{h}. \tag{3.3.15}$$

For any variable $z^i \in \mathbf{Z}$, equations (3.3.14) and (3.3.15) provide a lower bound on the fraction of the forecast error variance that can be associated with a specific transmission channel. While $\gamma_i(h)$ is a measurement at a specific forecast period h, $\bar{\gamma}_i$ is an average over the entire horizon of h periods.

So far, I have used the terms "channel" and "variable" interchangeably. In what follows, I allow a channel to be more general in the sense that it can encompass the transmission of monetary policy via a subset of variables. I define channel "x" of monetary policy transmission by considering the set of variables $\Gamma_{\text{x}} \subset \mathbf{Z}$ and letting

$$\text{FEV}^{\text{x}}_i(h) \equiv \sum_{z^k \in \Gamma_{\text{x}}} \text{FEV}_{i,k}(h). \tag{3.3.16}$$

Furthermore, I define the relative contribution of channel x to be given by

$$\phi^{\text{x}}_i(h) \equiv \frac{\text{FEV}^{\text{x}}_i(h)}{\text{FEV}^{\text{var}}_i(h)}, \text{ and} \tag{3.3.17}$$

$$\overline{\phi}^{\text{x}}_i \equiv \frac{\sum_{j=0}^{h} \phi^{\text{x}}_i(h)}{h}. \tag{3.3.18}$$

[60] From expressions (3.3.12) and (3.3.13), $\text{FEV}^{\text{var}}_i$ is the sum of the variances of the reduced form errors while $\text{FEV}^{\text{cov}}_i$ is the sum of the covariances of the reduced form errors. In both cases, the variances and covariances are associated with a one standard deviation of the structural monetary policy shock.

Expression (3.3.17) is a generalization of the ratio given by (3.3.10). That is, $\phi_i^x(h)$ quantifies, in percentage terms, the contribution of *channel* x for the monetary policy transmission at any forecast horizon $h \geq 1$. Note that (3.3.18) captures the same idea in terms of the average percentage contribution over the horizon h.[61]

3.4 Estimation

The following section describes the different VAR specifications I use. In addition, it discusses my choice of instruments for the identification of the monetary policy shock.

3.4.1 VAR specifications

I resort to three different VAR specifications: VAR_1, VAR_2, and VAR_3, each indlcuding twelve lags. The choice of these different specifications is motivated by the way in which I organize the paper. I proceed in three steps. First, I study the information content of the SLOOS. Second, I quantify the importance of the credit channel relative to that of the conventional channel. Lastly, within the credit channel, I quantify the contribution of the spreads and non-spreads sub-channels.

I use the VAR_1 to compare the information content between the SLOOS variables and the credits spreads. I take the baseline specification of Gertler and Karadi (2015) (henceforth VAR_{GK}) as my standard for comparison. The VAR_{GK} includes the log industrial production (IP), the log consumer price index (CPI), the one-year government bond rate (1YR), and three credit spreads: the excess bond premium (EBP), the mortgage spread (Mort_{SP}), and the commercial paper spread (CP_{SP}). Therefore, my VAR_1 includes the variables IP, CPI, 1YR and replaces the credit spreads with a subset of the principal components of the SLOOS variables (PC_{S}^1, PC_{S}^2, PC_{S}^3, PC_{S}^4, PC_{D}^1, PC_{D}^2, and PC_{D}^3). I choose the VAR_{GK} as the standard for two reasons. First, it is used in the study the monetary policy transmission via the credit channel, which is similar to my focus. Second, it uses the same identification scheme as I do. Therefore, I can easily compare my conclusions with any results derived using the VAR_{GK} and attribute the discrepancies to differences in the information content between the SLOOS and credit spreads rather than to identification assumptions.

[61]Note that given the definition of the ratios ϕ_i^x, it is not necessary to identify s_p^p. One can just estimate α (i.e. estimate $\mathbf{s}^p$ up to a scaling factor). Since the scaling factor is the same for all coefficients $s_i^p \in \mathbf{s}^p$, it doesn't affect the relevant ratio.

I use the other two specifications, VAR_2 and VAR_3, in order to take advantage of the information content of the SLOOS variables. The additional information of the SLOOS variables, which is not captured by the credit spreads, is mostly related to the credit conditions for household loans (mortgages, non-credit card and credit card loans).[62] Therefore, it is natural to think that this information is most relevant in the context of the transmission of monetary policy to measures of real aggregate activity pertaining to household consumption rather than those related to industrial production. Hence, instead of the IP variable, I use the log consumer durable expenditures (D), the log consumer non-durable expenditures (C), and the log households' debt (B) in the VAR_2 and VAR_3 specifications.

More specifically, I use the VAR_2 to quantify the contribution of the conventional and credit channels for the monetary policy transmission mechanism. To this end, I consider three different variants of the VAR_2, which differ on their credit block. The first variant uses the three credit spreads (EBP, Mort_{SP}, and CP_{SP}); the second uses the principal components of the credit supply and demand indicators (PC^1_{S}, PC^2_{S}, PC^3_{S}, PC^4_{S}, PC^1_{D}, PC^2_{D}, and PC^3_{D}); and the third one omits credit variables altogether. By using the different variants I can assess whether the conventional and credit channels' contribution differs substantially when using alternative measures of credit conditions. The instance without credit variables is used as a counterfactual experiment to analyze whether or not the conventional channel is over estimated in a misspecified VAR that completely shuts down the credit channel.

Finally, I use the VAR_3 specification in order to asses if, within the credit channel, the non-spread sub-channel is important for the monetary policy transmission. Taking advantage of the larger information content of the SLOOS variables, I include in the credit block the three credit spreads (EBP, Mort_{SP}, and CP_{SP}) and the principal components of the SLOOS variables that reflect the credit conditions for household loans (PC^2_{S}, PC^3_{S}, PC^4_{S}, PC^1_{D}, and PC^3_{D}). These principal components contain information about the credit markets which is not reflected by the credit spreads; hence they represent the non-spread sub-channel within the credit channel. Given this setting, I can quantify and assess the importance of both, spread and non-spread, mechanisms for the transmission of monetary policy.

[62] See Section 3.2.

3.4.2 Instrument Choice

As per the instrumental variables methodology, any valid set of external instruments must satisfy two conditions: relevance and exogeneity. In the context of the model that I consider, these two conditions are given by equations (3.3.4) and (3.3.5). The former amounts to the requirement that the external instruments must be sufficiently correlated with the fundamental monetary policy shock. The latter implies that the external instruments need to be uncorrelated with all the other fundamental disturbances implied by the model.[63]

Following the literature on High Frequency Identification (HFI) of monetary policy shocks, I consider the set of futures rates surprises on FOMC meeting dates, as proposed by Gürkaynak, Sack and Swanson (2005), as my candidate set of instruments. This set includes the surprises in the current and three month ahead monthly fed funds futures (FF_0 and FF_3, respectively) and the surprises in the six, nine and twelve month ahead futures on three month Eurodollar deposits (ED_6, ED_9, and ED_{12}, respectively). This instrument data is available at a monthly frequency for the period 1991:1 through 2012:6, which roughly coincides with the period for which the SLOOS data is available.[64]

Each of the future rate surprises is constructed as the difference between the settlement price of the futures contract on the FOMC meeting day and the corresponding settlement price on the previous day. Note that these instruments are constructed to meet the relevance and exogeneity conditions. First, they reflect news only about monetary policy while being uncontaminated with news about other unobserved fundamental shocks to the economy. This is ensured by taking the price measurements only within a 30 minute window of the FOMC announcement. Second, they are a strong proxy for the changes in rates due only to unanticipated monetary policy news. This is ensured by measuring the surprises using future rate contract prices as opposed to using differences between realized and forecasted rates. The latter can be potentially affected by other unobserved factors, such as the risk premium. Provided that, for any of the futures contracts considered here, the risk premium (or any other unobserved factor) remains constant during the 24 hour period preceding the FOMC meeting, the changes in rate futures isolate the effect of unanticipated monetary policy

[63]For a more detailed discussion about the choice of external instruments within the High Frequency Identification literature, refer to Stock and Watson (2012), Mertens and Ravn (2013), and Gertler and Karadi (2015).

[64]The original set of instruments was constructed at a daily frequency. In order to use it in a VAR setting, Gertler and Karadi (2015) use a weighting scheme to aggregate it to a monthly frequency, which is the data I use.

news.[65]

From within this set of instruments, ideally, I would like to select one that captures a certain degree of forward guidance. Unanticipated monetary policy news are relevant not only to the extent to which they affect the current short term rates, but also due to their effect on the expected *future* path of short term rates. Within the HFI framework, the persistent effect of unanticipated monetary policy news can be captured in two ways. One is by measuring the effect of unanticipated monetary policy news on a mid- or long-term rate which, by a simple term structure argument, reflects the future path of short-rates. Another one is by using instruments that directly reflect expectations of short rate movements further in the future. From this perspective, the surprises in future contracts settled three, six, nine or twelve months ahead are better suited as external instruments than the surprises in current futures.[66]

However, using an instrument that captures forward guidance is not always feasible in light of an additional restriction I face. Given that part of my procedure requires comparing VAR specifications that use different measures of credit costs, I need to ensure that the identification procedure is consistent across them. If this is not the case, the differences in the predictions of the VAR's might stem from identification issues rather than differences in the information content of the alternative measures. I define an instrument to be consistent when it satisfies two criteria. First, the weak instrument problem can be safely ruled out when using *both* sets of credit measures. For ruling out a weak instrument problem, I use the criteria proposed by Stock, Wright and Yogo (2002), who suggest a threshold value of ten for the first-stage regression robust F-statistic. Second, the explanatory power of the instrument for the VAR residual of the monetary policy indicator must be sufficiently similar under the credit spreads and the SLOOS variables.

With these considerations in mind, I contemplate using the surprise in the three month ahead fed funds future (FF_3) as the instrument and the one-year government bond rate (1YR) as the monetary policy indicator.[67] Table 3.3 summarizes some statistics for the regression of the VAR residuals

[65]For more on this, see Kuttner (2001), Piazzesi and Swanson (2008), and Hamilton (2009).

[66]Evidence that the persistent effect of monetary policy news is captured better by mid-term bond rates instrumented by surprises in futures contracts settled further in the future than by short-term rates instrumented by surprises in current future contracts is provided by Kuttner (2001), Bernanke, Reinhart and Sack (2004), Swanson and Williams (2014), among others.

[67]Since I use the baseline VAR from Gertler and Karadi (2015) as the standard for assessing the informational content of the SLOOS, I try to remain as close as possible to their estimation procedure; which uses this combination of policy indicator and instrument.

Table 3.3: Effects of High-Frequency Instrument on the First Stage Residuals of the Monetary Policy Indicator for the Different VAR Specifications

	VAR_{GK}	$\widehat{VAR}_{GK}$	VAR_1	$\widehat{VAR}_1$
Constant	0.010	0.010	0.009	0.010
	(0.006-0.014)	(0.007-0.013)	(0.005-0.012)	(0.006-0.014)
FF_3	0.800	0.785	0.683	0.800
	(0.459-1.071)	(0.480-0.981)	(0.343-0.926)	(0.439-1.065)
Observations	258	258	258	258
Adj. R^2	0.065	0.065	0.050	0.065
F-statistic	16.713	21.744	11.510	16.713

		VAR_2		
	Spreads	SLOOS	No Credit	
Constant	0.007	0.006	0.008	
	(0.004-0.010)	(0.003-0.009)	(0.002-0.011)	
FF_3	0.524	0.439	0.579	
	(0.239-0.716)	(0.200-0.522)	(0.185-0.881)	
Observations	258	258	258	
R^2	0.031	0.038	0.028	
F-statistic	9.892	11.597	7.273	

	VAR_3			
	FF_3	FF_0		
Constant	0.005	0.005		
	(0.002-0.008)	(0.003-0.008)		
Instrument	0.364	0.318		
	(0.134-0.440)	(0.137-0.388)		
Observations	258	258		
R^2	0.033	0.036		
F-statistic	8.463	11.007		

Note: Sample period 1990:1-2012:6. 90 percent confidence intervals in parenthesis.
1YR is used as the monetary policy indicator.
FF_3 is used as the instrument.
VAR_1 specification includes PC_S^1, PC_S^2, and PC_S^4

for the 1YR on the FF_3. The statistics are presented for each of the eight VARs that I use for my analysis: two versions of the VAR_{GK}, two versions of the VAR_1, the three variants of the VAR_2 (one with no credit variables -No Credit-, one with the credit spreads -Spreads-, and one using the orthogonalized SLOOS variables -SLOOS-), and the VAR_3. The total sample size, adjusted R^2 and robust F-statistic are reported at the bottom for each VAR specification.

For the VAR_{GK} and the VAR_1, all four variants have a robust F-statistic that is larger than ten, suggesting that a weak instrument problem can be ruled out. In addition, the explanatory power of the instrument is uniform across all of these variants. The FF_3 explains about 6.5 percent of the monthly innovation in the one-year rate when the credit costs are measured using the spreads and about 5 percent when the SLOOS variables are used instead.[68] Therefore, given that the FF_3 is consistent, I can safely use it as the instrument for the first part of my study.

For the VAR_2, the FF_3's explanatory power is uniform across all three variants. It explains about 3 percent of the monthly innovation in the one year rate when credit costs are either ignored altogether or measured using the credit spreads; while it explains about 4 percent when the SLOOS variables are used instead. However, the only variant that has a robust F-statistic larger than ten is the one that uses the SLOOS variables. For the other two specifications, I can't rule out the weak instrument problem. Nevertheless, if I use the surprise in the current fed funds future (FF_0) as the instrument instead of the surprise in the three months ahead fed funds future (FF_3), all three variants of the VAR_2 have F-statistics larger than ten and the weak instrument problem can be ruled out. In addition, the FF_0 instrument explains about 4 to 5 percent of the monthly innovation in the 1YR for all three variants. That is, the performance of the FF_0 is consistent across all three instances of the VAR_2. Therefore, for the second part of my study, I use the FF_0 instead of the FF_3 as the instrument in order to ensure a consistent identification procedure.[69]

Finally, for the VAR_3, the table presents the first stage regression results under both instruments, FF_3 and FF_0. The FF_3 explains about 3 percent of the monthly innovation in the one year rate. However, since the robust F-statistic is only about eight, the weak instrument problem can't be ruled out. On the other hand, the FF_0 has a similar explanatory power but a robust F-statistic that is above the threshold of ten. Therefore, for the third part of my study, I choose to instrument the VAR_3 with the FF_0.[70]

[68]This result is in line with Gertler and Karadi (2015), who find an explanatory power of about 7.8 percent for the FF_3. The slight discrepancy comes from the difference in sample periods.

[69]If the weak instrument problem is ignored and FF_3 is used as the instrument, the nature of the results derived from the VAR_2 specifications remains unchanged. If anything, the relevance of the credit channel relative to the conventional channel increases.

[70]As Table 3.3 suggests, the performance of both instruments (ignoring the weak instrument problem) is very similar under the VAR_3. Therefore, it is no surprise that the results about the composition of the credit channel (the contribution of the spread and non-spread sub-channels) remain unchanged when using either of the two instruments.

In conclusion, I use two instruments for the identification of the monetary policy shock. On one hand, I use the surprise in the three months ahead fed funds future (FF_3) as the instrument to identify the monetary policy shock when comparing the information content between the SLOOS variables and the credit spreads. On the other hand, when quantifying the contribution of the credit channel and its sub-channels for the monetary policy transmission, I use the surprise in the current fed funds future (FF_0) as the instrument. This choice of instruments not only guarantees that the exogeneity and relevance conditions hold, but it also ensures two additional properties. First, the possibility of a weak instrument problem can be safely ruled out. Second, the instruments' performance (in terms of explanatory power) is consistent when using different variables to measure the credit conditions (either the spreads or the SLOOS variables). Hence, when comparing the predictions of the VARs under alternative credit measures, any differences can be attributed to the actual monetary policy propagation mechanism rather than to disparities in the identification procedure.

3.5 Results

In what follows, I present the three main results of the paper. First, the SLOOS variables not only reflect the same information as the credit spreads, but also some additional information which is relevant for monetary policy transmission. Second, the credit channel is as important as the conventional channel. It can account for a large portion (between 15% to 30%) of the variance in households' durable and non-durable expenditures following an unexpected monetary policy announcement. Finally, non-spread credit factors play a significant role for monetary policy transmission. Roughly speaking, non-spread factors account for at least half of the credit channel's contribution to the variance of durable and non-durable expenditures.[71]

3.5.1 Information content on the SLOOS

To compare the information content between the SLOOS variables and the credit spreads I proceed in two steps. First, I use the SLOOS data to try to reproduce the impulse response functions (IRF_{GK}) implied by the VAR_{GK}. I construct the linear projections of the three credit spreads onto the principal components of the SLOOS variables and include them as the credit block in the modified version

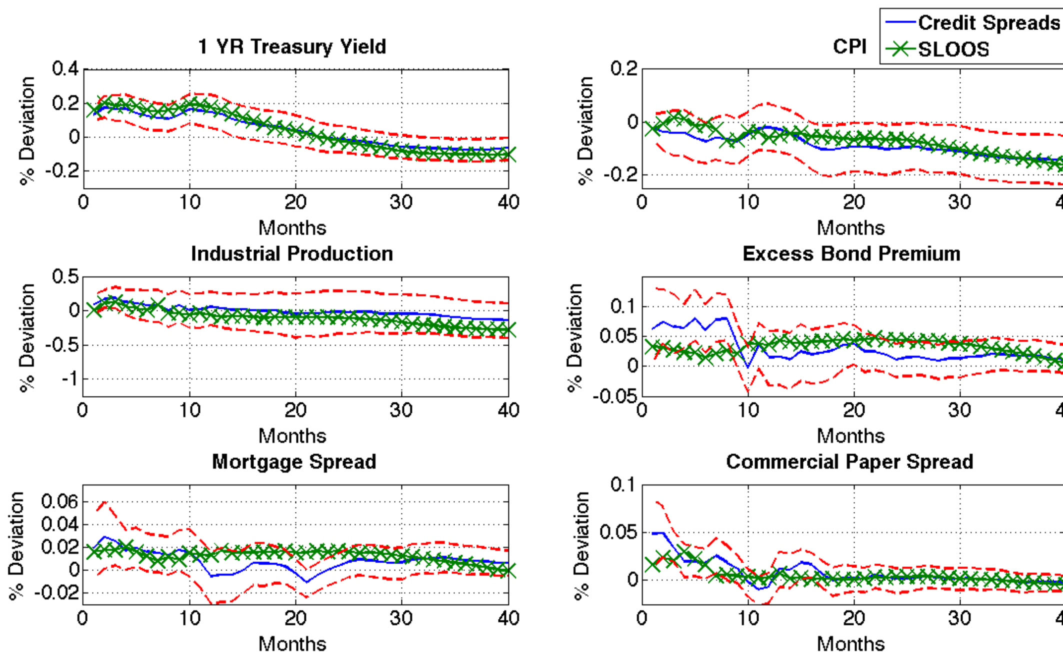
[71]Examples of such non-spread factors are credit limits, collateral requirements, credit score requirements, just to name a few.

$\widehat{\mathrm{VAR}}_{\mathrm{GK}}$. I use this modified version to estimate impulse responses, $\widehat{\mathrm{IRF}}_{\mathrm{GK}}$, which I then compare with the original set of responses, $\mathrm{IRF}_{\mathrm{GK}}$. Second, I use the credit spreads to try to reproduce the impulse response functions (IRF_1) implied by the VAR_1. As in the previous step, I construct a vector of linear projections of the principal components onto the credit spreads. I then construct the modified version $\widehat{\mathrm{VAR}}_1$ by including the projected principal components as the credit measures. I use this modified version to estimate impulse responses, $\widehat{\mathrm{IRF}}_1$, which I compare with the original set of responses IRF_1. The goal of this thought experiment is to understand if, in the context of monetary policy transmission, the information set of the credit spreads is a subset of that of the SLOOS variables and vice-versa.

The simple linear regression analysis of Section 3.2 already provides evidence of the larger information content of the SLOOS variables relative to the credit spreads. However, it remains to show that this additional information is relevant for the monetary policy transmission mechanism. As a first step towards this goal, consider the results from Table 3.3 in Section 3.4.2, which pertain to the identification of the monetary policy shock. The first stage regression results are nearly identical for the $\mathrm{VAR}_{\mathrm{GK}}$ and $\widehat{\mathrm{VAR}}_{\mathrm{GK}}$ specifications. This is true not only in terms of the explanatory power of the FF_3 instrument, but also in terms of the point estimates for the regression coefficients. This shows that the results obtained using the credit spreads can be reproduced using an appropriate combination of the SLOOS variables instead. On the other hand, the outcome of the first stage regression for the VAR_1 is rather different than that of the $\widehat{\mathrm{VAR}}_1$. That is, the results obtained using the SLOOS variables can't be reproduced using the credit spreads. Furthermore, under the $\widehat{\mathrm{VAR}}_1$, the regression coefficient estimates, the adjusted R^2, and the robust F-statistic are all nearly identical to those obtained using the $\mathrm{VAR}_{\mathrm{GK}}$. In other words, the projection of the SLOOS variables onto the credit spreads and the credit spreads themselves seem to span the same space, at least in terms of the information relevant for the identification of the monetary policy shock. Altogether, these three observations would suggest that, in the context of monetary policy transmission, the information contained in the credit spreads is a subset of the information contained by the SLOOS variables.[72]

[72]Further evidence is provided in Table 5.3 (in Section **??**). In line with the conclusions at the end of Section 3.2, the results from Table 5.3 suggest that the information contained in the credit spreads is entirely captured in only two of the principal components ($\mathrm{PC}_{\mathrm{S}}^1$ and $\mathrm{PC}_{\mathrm{D}}^2$) of the SLOOS data. These two components are labeled as the "main" components of the credit supply and demand blocks, respectively.

Figure 3.2: A Surprise Monetary Tightening
VAR_{GK} vs. $\widehat{\text{VAR}}_{\text{GK}}$



Note: VAR_{GK} includes the three credit spreads (EBP, Mort_{SP}, and CP_{SP}).
90 percent confidence bands given by dashed lines.
Responses are based on a one standard deviation of the fundamental monetary policy disturbance.

As for the the impulse response functions, Figure 3.2 shows the IRF_{GK} (the solid blue line labeled "Credit Spreads", with 90 percent confidence bands given by the dashed red lines) and the $\widehat{\text{IRF}}_{\text{GK}}$ (the green line labeled "SLOOS") following a surprise monetary policy tightening. The IRF_{GK} show that, consistent with conventional theory, the one-year rate increases by about 20 basis points upon impact and remains significantly positive for roughly a year. Although there is a decrease in the consumer price index, it is not significant (in the statistical sense) until after a year and a half. This might be a consequence of the sluggishness in price adjustments. Note that contrary to standard theory, there is not a significant decline in industrial production. This is not surprising however, as there are other empirical studies, such as Boivin, Kiley and Mishkin (2010), that have argued that in recent decades monetary policy innovations have a more muffled effect on real activity.[73] Regarding the credit spreads, the responses in Figure 3.2 are in line with the findings of Gertler and Karadi (2015). All three spreads increase for a period of about eight months before

[73]The authors in Gertler and Karadi (2015) find that the monetary policy tightening does lead to a significant decrease in industrial production. I attribute this difference to the sample period they use (1979:7-2012:6) as opposed to the one I consider (1990:1-2012:6).

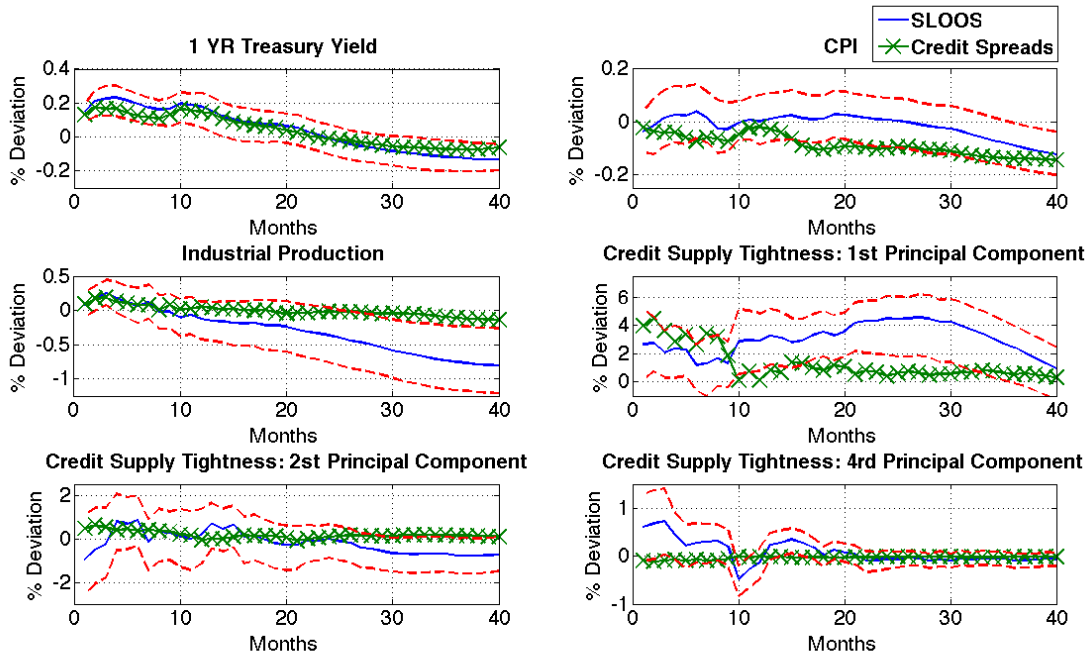
returning to their trend levels. The excess bond premium increases about 7.5 basis points, remains at that level during the first eight months, and then suddenly reverts back to trend. The increase in the mortgage spread is more modest, only about 2 basis points, and displays the same pattern as the excess bond premium.[74] The commercial paper spread increases about 5 basis points upon impact for roughly two months, then it decreases to about 2 basis points for another 6 months, and then it finally reverts back to trend.

It is evident from Figure 3.2 that the $\widehat{\mathrm{IRF}}_{\mathrm{GK}}$ for the one-year rate, the consumer price index, and the industrial production are well within the 90 percent confidence bands of their $\mathrm{IRF}_{\mathrm{GK}}$ counterparts (indeed, they are almost identical). However, this is not the case for the three credit spreads. The SLOOS variables are unable to capture the initial increase in the excess bond premium and in the commercial paper spreads. For the excess bond premium, they predict an increase of only about 2.5 basis points in the initial eight months when the actual increase is of 7.5 basis points. For the commercial paper spread, they can only account for a 2 basis point increase upon impact while the actual increase is of about 5 basis points. For the mortgage spread, although the SLOOS variables are able to capture the increase of about 2 basis points for the eight month period after impact, they overestimate the persistence of such increase. Overall, these results suggest that the information in the credit spreads, at least the one relevant for monetary policy transmission to *aggregate economic activity*, is also contained within the SLOOS variables.

Figure 3.3 presents the responses for the specification $\mathrm{VAR}_{\mathrm{S},1}$, in which the credit costs are measured using three of the principal components of the credit supply block. The $\mathrm{IRF}_{\mathrm{S},1}$ (solid blue line labeled "SLOOS" with 90 percent confidence bands given by the dashed red lines) and the $\widehat{\mathrm{IRF}}_{\mathrm{S},1}$ (the green line labeled "Credit Spreads") are given following an unanticipated monetary policy tightening. The tightening induces an increase in the one-year rate of about 20 basis points upon impact and then it reverts back to trend after a year. The consumer price index remains at its trend value for most of the horizon, decreasing only after about 38 months. Consistent with standard theory, the industrial production decreases. However, the decrease in economic activity occurs with a lag (it becomes statistically significant after roughly two years). The responses of the principal

[74]The response of the mortgage spread is different than the one presented in Gertler and Karadi (2015). In their case, the mortgage spread increases about 2 basis point upon impact, then increases sharply to 7 basis points after two months, and finally it slowly decreases until it reverts back to trend after about 8 months. Again, I attribute this discrepancy to the different sample periods between my study (1990:1-2012:6) and theirs (1979:7-2012:6). In particular, the bulk of my sample period consists of housing boom years, where mortgage spreads remained low for different reasons.

Figure 3.3: A Surprise Monetary Tightening
$\text{VAR}_{S,1}$ vs. $\widehat{\text{VAR}}_{S,1}$



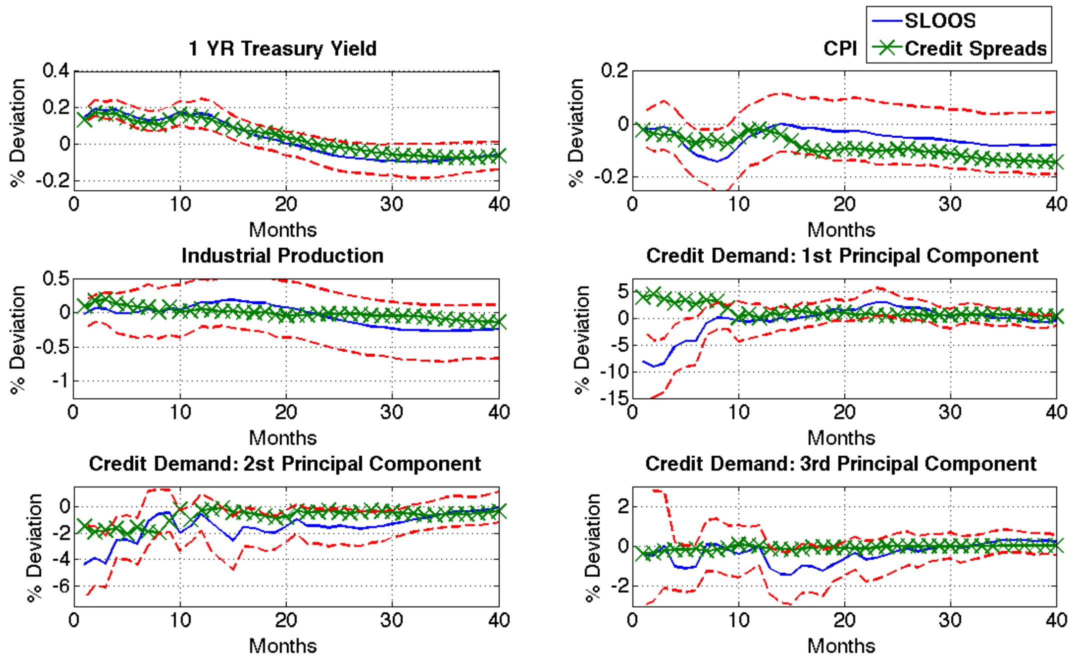
Note: $\text{VAR}_{S,1}$ includes the 1st, 2nd, and 4th principal components of credit supply variables (PC_S^1, PC_S^2, and PC_S^4).

90 percent confidence bands given by dashed lines.

Responses are based on a one standard deviation of the fundamental monetary policy disturbance.

components of the credit supply block point to a general tightening of the credit conditions. The first and fourth principal components both experience a significant increase. The former, which is the one most closely related to the credit spreads, increases upon impact and remains above trend for about 38 months. The latter, which is not correlated with credit spreads and thus measures other types of credit costs, increases upon impact and remains above trend for about 20 months. The second principal component does not significantly deviate from its trend during the 40 month horizon. Note that the responses presented in Figure 3.3 differ considerably from those in Figure 3.2. Although the responses for the one-year rate are similar, this is not the case for the rest of the variables. The most significant difference is perhaps the larger persistence in the tightening of the credit conditions when using the SLOOS variables. In Figure 3.2, the credit conditions tighten upon impact for a period of about 8 months. However, in Figure 3.3, the first and fourth principal components show a tightening of the credit conditions for a much larger horizon, at least 20 months.

Figure 3.4: A Surprise Monetary Tightening
$\mathrm{VAR}_{\mathrm{D},1}$ vs. $\widehat{\mathrm{VAR}}_{\mathrm{D},1}$



Note: $\mathrm{VAR}_{\mathrm{D},1}$ includes the 1st, 2nd, and 3rd principal components of credit demand variables (PC_D^1, PC_D^2, and PC_D^3).
First-stage regression Robust F-stat: 20.68.
90 percent confidence bands given by dashed lines.
Responses are based on a one standard deviation of the fundamental monetary policy disturbance.

This persistence of the credit tightening following the unanticipated monetary innovation might be responsible for explaining the lagged decrease in the industrial production.

It is clear that the $\widehat{\mathrm{IRF}}_{\mathrm{S},1}$ in Figure 3.3 do not reproduce the $\mathrm{IRF}_{\mathrm{S},1}$ as closely as in the previous comparison. Although the credit spreads can reproduce the results obtained with the SLOOS for the one-year rate, they fail in reproducing the response for the consumer price index and the industrial production. Furthermore, the credit spreads are unable to reproduce the response for the credit supply principal components. Most notably, the credit spreads fail to capture the persistent increase in the first principal component and the tightening of the fourth principal component during the initial eight months. I interpret this as evidence that there is additional information in the SLOOS credit supply variables, which is relevant for monetary policy transmission to *aggregate economic activity*, that is not captured by the credit spreads.

The impulse responses for an alternative specification ($\text{VAR}_{\text{D},1}$), in which the credit costs are measured using the three principal components of the SLOOS credit demand block, are presented in Figure 3.4. As in the previous cases, the $\text{IRF}_{\text{D},1}$ (solid blue line labeled "SLOOS" with 90 percent confidence bands given by the dashed red lines) and the $\widehat{\text{IRF}}_{\text{D},1}$ (the green line labeled "Credit Spreads") are given following a surprise monetary tightening. Note that the responses for the one-year rate, the industrial production, and the consumer price index, are similar to those in Figure 3.2. The response of the demand principal components suggests a credit contraction following the first 8 months after impact. Interestingly, this contraction in the credit demand parallels the increase in the credit spreads given in Figure 3.2. Altogether, this suggests that the short term changes in credit conditions relevant for the monetary policy transmission are captured by both, the credit spreads and the credit demand block of the SLOOS.

As it can be seen from Figure 3.4, the $\widehat{\text{IRF}}_{\text{D},1}$ closely resemble the $\text{IRF}_{\text{D},1}$ for the one-year rate, the industrial production, and the consumer price index. In this sense, the credit spreads do a better job reproducing the results obtained using the SLOOS credit demand variables relative to those obtained using the SLOOS credit supply block. This is consistent with the previous observation that much of the relevant information about the short term changes in credit conditions is contained in both, the credit spreads and in the demand block of the SLOOS. However, note that the information contained in the credit spreads does not suffice to reproduce the individual responses of the credit demand principal components. In particular, the credit spreads are only able to capture the initial demand contraction for the second principal component. They fail to capture the initial demand contraction for the first principal component or any of the dynamics for the third principal component.

The previous analysis suggests that, in the context of monetary policy transmission, the information set of the credit spreads is a subset of the SLOOS information set. The additional information of the SLOOS variables originates from two sources. First, following an unanticipated policy shock, the credit spreads capture only short term changes in the credit conditions. The SLOOS variables capture both, short and long term changes. The short term changes are reflected mostly in the credit demand block while the long term changes are captured by the credit supply block. Second, the information in the credit spreads reflects a "common" component of the conditions in all credit markets, which is strongly influenced by the conditions associated with the commercial

Table 3.4: $\bar{\gamma}_i$ Under the Different VAR_2 Specifications

	12 Month Horizon			24 Month Horizon			36 Month Horizon		
	Spreads	SLOOS	No Credit	Spreads	SLOOS	No Credit	Spreads	SLOOS	No Credit
1 YR Yield	81.5	92.4	97.9	75.9	92.4	97.9	72.9	90.4	96.9
CPI	78.4	92.7	61.2	77.2	89.2	58.0	80.1	87.0	56.3
Durables	80.2	78.5	74.5	83.9	75.2	65.2	82.2	81.5	61.5
Non-Durables	70.3	62.3	56.6	76.0	64.9	54.4	77.3	71.6	53.8
HH Debt	56.5	55.5	56.0	60.6	53.5	53.6	68.4	53.9	53.5

Note: Results are based on a one standard deviation of the monetary policy shock.

and industrial loans. Consequently, the credit spreads can only account for the dynamics of two (of the seven) principal components of the SLOOS variable: PC_S^1 and PC_D^2. The remaining principal components, which reflect the credit demand and supply conditions for household loans (i.e. mortgage, credit card and non-credit card), contain information that is not reflected by the credit spreads. Importantly, these non-spread components are significantly affected by a surprise monetary policy innovation. Therefore, for the study of monetary policy transmission to macroeconomic indicators that pertain to household consumption, such as durable and non-durable expenditures, using the credit spreads might ignore some, potentially relevant, channels of transmission of monetary policy. This is the issue to which I turn next.

3.5.2 Transmission of Monetary Policy

The results I present are obtained using the VAR_2 specifications instrumented with the surprise in the current monthly fed funds future (FF_0).[75] To quantify the contribution of the different channels for the monetary policy transmission, I use $\bar{\gamma}_i$ and $\bar{\phi}_i^x$ as defined in (3.3.15) and (3.3.18). Intuitively, given the total variance of variable i that results from an unexpected monetary policy announcement, $\bar{\gamma}_i$ quantifies the fraction of this variance that can be attributed to distinct channels of transmission. From within this fraction, $\bar{\phi}_i^x$ quantifies the contribution of a particular channel (channel x) of interest.

Table 3.4 shows $\bar{\gamma}_i$ for each of the three different VAR_2 specifications for forecast horizons of one, two, and three years. For the two specifications that include credit variables (Spreads and

[75]All results still hold when the three month ahead fed funds future (FF_3) is used as the instrument. Furthermore, for the real activity sector of the economy (household's debt, durable and non-durable expenditures), using FF_3 as the instrument increases the fraction of the monetary policy disturbance that is transmitted via the credit channel. Hence, when the instrument incorporates some forward guidance, the credit channel becomes more relevant.

SLOOS), these results suggest that at least 54% of the variation in households' debt, 62% of the variation in non-durable expenditures, and 75% of the variation in durable expenditures can be attributed to specific transmission channels. For the specification without credit variables, the fraction of the variance for these three variables that can be attributed to specific transmission channels is substantially smaller. To see why this is the case, suppose that some of the variation in the variables (1YR, CPI, D, C, B) is transmitted via the credit variable y. When y is omitted, this variation is attributed to the contemporaneous interaction between these variables, rather than to the common factor y. Therefore, as the this variation can't be associated with one particular transmission channel it is not included in $\bar{\gamma}_i$.

Given that a reasonable amount of the variation in the VAR_2 variables can be associated with particular transmission channels, I next proceed to quantify the individual contribution of these channels. Specifically, I am interested in measuring the contribution of four channels: price/rate (PR), spreads (SP), credit supply conditions (PC_S), and credit demand conditions (PC_D). Hence, in the definitions (3.3.16), (3.3.17), and (3.3.18), $x \in \{\text{PR}, \text{SP}, \text{PC}_\text{S}, \text{PC}_\text{D}\}$. For the price/rate channel, I let $\Gamma_{\text{PR}} = \{1\text{YR}, \text{CPI}\}$. I use the PR channel as a proxy for the conventional channel as it captures the effect of the unanticipated monetary policy news via contemporaneous price and one-year rate adjustments. As for the spreads channel, I let $\Gamma_{\text{SP}} = \{\text{EBP}, \text{Mort}_{\text{SP}}, \text{CP}_{\text{SP}}\}$ for the variant with the credit spreads and $\Gamma_{\text{SP}} = \left\{\text{PC}_\text{S}^1, \text{PC}_\text{D}^2\right\}$ for the specification with the SLOOS variables. Therefore, the spreads channel captures the effect of the unanticipated policy news via contemporaneous adjustments in spreads.[76] The remaining two channels only apply to the VAR_2 specification that uses the SLOOS credit variables. For the credit supply channel, I let $\Gamma_{\text{PC}_\text{S}} = \left\{\text{PC}_\text{S}^2, \text{PC}_\text{S}^3, \text{PC}_\text{S}^4\right\}$. The PC_S channel then captures the effect of the unexpected monetary policy news via contemporaneous changes in credit supply conditions *other* than spreads. This is because the credit supply principal components included in this channel are those orthogonal to the credit spreads. Finally, for the credit demand channel, I let $\Gamma_{\text{PC}_\text{D}} = \left\{\text{PC}_\text{D}^1, \text{PC}_\text{D}^3\right\}$. The PC_D channel encompasses the effect of unexpected monetary policy news via contemporaneous changes in the demand for credit. This channel is an indirect proxy for credit costs and conditions via credit demand. In this sense, a priori, the PC_D channel can't be exclusively associated with neither the conventional channel nor the credit

[76] For the specification with the SLOOS variables, I use the "main" principal components, PC_S^1 and PC_D^2, in the definition of the SP channel given that these components are directly related to the credit spreads, as pointed out in Sections 3.2 and 3.5.1.

Table 3.5: Contribution of Different Channels to the Monetary Policy Transmission Under the VAR_2

		Price/Rate			Spreads		Credit Supply	Credit Demand
		Spreads	SLOOS	No Credit	Spreads	SLOOS	SLOOS	SLOOS
12 Months	1 YR Yield	98.0	88.4	99.8	1.4	0.9	0.8	1.5
	CPI	84.3	78.1	92.8	8.6	7.3	3.7	5.5
	Durables	19.1	31.7	19.1	8.1	9.4	3.5	2.9
	Non-Durables	43.6	39.3	67.8	34.7	6.8	1.1	28.3
	HH Debt	32.3	40.3	35.3	6.5	3.2	0.5	3.1
24 Months	1 YR Yield	95.4	81.8	99.5	3.1	2.4	1.2	1.6
	CPI	74.4	73.8	86.0	15.4	10.9	4.2	5.8
	Durables	31.5	36.3	30.1	13.6	14.6	3.8	4.0
	Non-Durables	40.6	40.8	61.8	39.9	17.0	3.0	20.6
	HH Debt	41.5	46.5	45.0	9.9	2.7	0.5	3.9
36 Months	1 YR Yield	93.3	78.2	99.2	4.2	3.3	1.3	1.7
	CPI	65.4	67.9	81.6	23.1	16.6	4.6	5.5
	Durables	39.0	45.1	37.4	16.0	13.9	3.6	3.8
	Non-Durables	41.4	43.5	60.2	40.4	19.7	4.5	16.4
	HH Debt	49.8	48.8	50.6	11.1	2.5	0.5	3.7

Note: The contribution of the each channel is measured by $\bar{\phi}_i^x$ and is based on a one standard deviation of the monetary policy shock. The three different VAR_2 specifications are:
(a) Spreads: credit costs measured using the EBP, mortgage and commercial paper spreads.
(b) SLOOS: credit costs measured using the SLOOS credit supply/demand principal components.
(c) No Credit: no credit variables included in the VAR.

channel (spread and non-spread). This is because the credit costs might include the term structure of interest rates (conventional channel), as well as credit spread and non-spread factors. However, in the definition of Γ_{PC_D}, the included demand principal components are those orthogonal to the credit spreads. That is, I am effectively removing the credit spread component out of credit costs that are being indirectly proxied by the PC_D channel. Thus this channel is an indirect proxy of the conventional channel and the non-spread component of the credit channel only.

With these definitions at hand, I can evaluate the importance of the credit and conventional channels for the monetary policy transmission. In all three specifications of the VAR_2, I use the PR channel as the proxy for the conventional channel. The credit channel, on the other hand, is specification-dependent. For the specification that uses the credit spreads (Spreads), the credit channel is just given by the SP channel; meanwhile, for the specification that uses the SLOOS variables (SLOOS), the credit channel is jointly given by the SP and PC_S channels. I do not include the PC_D channel as part of the credit channel because, as stated before, this channel proxies a mixture of the conventional and non-spread credit channels, which I can not disentangle in the current setting. Table 3.5 presents the contribution of the price/rate (PR), spreads (SP), credit supply ($\mathrm{PC_S}$), and credit demand ($\mathrm{PC_D}$) channels to the monetary policy transmission for some selected VAR_2 variables over horizons of one, two, and three years. The price/rate channel can be measured for all three specifications, the spreads channel can only be measured for the two specifications that include a credit block, and the credit supply/demand channels can only be measured for the specification that uses the SLOOS as the credit block.

For the Spreads specification, the results in Table 3.5 suggest that the contribution of the credit channel for the monetary policy transmission is between $6\% - 11\%$ for households' debt, $8\% - 16\%$ for durable expenditures, and $35\% - 40\%$ for non-durables expenditures. On the other hand, the conventional channel's contribution is around $32\% - 50\%$ for households' debt, $19\% - 39\%$ for durables, and $41\% - 44\%$ for non-durables. Overall, the conventional channel seems to be more relevant for the monetary policy transmission to most measures of real activity. Its contribution is about five times larger than that of the credit channel for households' debt and about twice the size for durable expenditures. However, for non-durable expenditures, both channels are roughly of the same size.

For the SLOOS specification, the contribution of the credit channel is between $3\% - 3.7\%$ for households' debt, $13\% - 18\%$ for durables, and $8\% - 24\%$ for non-durables. The bulk of the credit channel's contribution comes from the spread sub-channel, which is between three to five times larger than the credit supply (non-spread) sub-channel. As for the conventional channel, its contribution is around $40\% - 49\%$ for households' debt, $32\% - 45\%$ for durables, and $39\% - 44\%$ for non-durables. Hence, ignoring the contribution of the credit demand channel in the SLOOS specification has two important implications. First, the conventional channel is significantly more relevant than the credit channel for all three measures of aggregate activity. Second, within the credit channel, the non-spread sub-channel's contribution is rather small.

Comparing the results under the Spreads and SLOOS specifications, it is clear that the contribution of the conventional channel is robust to the choice of credit variables while that of the credit channel is not. For the latter, both specifications agree on the size of its contribution for durable expenditures only. For non-durable expenditures and households' debt, under the SLOOS, the size of the credit channel is between two and four times smaller than under the Spreads. From Table 3.5, this discrepancy seems to stem from changes in the composition of the credit channel. Recall that the information contained in the commercial paper spread (CP_{SP}) is mostly explained by the credit demand variables.[77] Consistent with this observation, at least for non-durable expenditures and household's debt, a large portion of the transmission through the credit spreads channel takes place via the CP_{SP}. Therefore, in the SLOOS specification, this effect is captured by the credit demand variables and only very slightly reflected in the credit supply ones. Thus if, in the SLOOS specification, the credit channel is redefined as the sum of the spreads, credit supply and credit demand sub-channels, its size becomes roughly the same as under the Spreads specification.[78] Additionally, if the credit channel is redefined in this way, the contribution of the credit spread sub-channel decreases, accounting for only about 40% and 60% of the credit channel (depending on the variable under consideration).

In addition, the results for the No Credit specification in Table 3.5 suggest that the exclusion of credit variables leads to a biased estimation of the contribution of the conventional channel. Although the size of the conventional channel is roughly the same under the No Credit and the Spreads/SLOOS specifications for durable expenditures and households' debt, this is not the case for non-durable expenditures. Under the No Credit specification, roughly 60% of the transmission of policy to non-durable expenditures takes place via the Price/Rate channel. In contrast, under either the SLOOS or Spreads, the contribution of this channel is only about 40%. This suggests that when the credit conditions are ignored altogether, the importance of the conventional channel is significantly overemphasized for non-durable expending.

To summarize, there are three main results from this analysis. First, the exclusion of credit variables in the model leads to an overestimation of the conventional channel. Second, the credit channel plays an important role in the monetary policy transmission for non-durable and durable expenditures. Particularly, for non-durable expenditures, the credit channel's contribution is about 30%; which is as large as the conventional channel's contribution. Third, the composition of the credit channel is relevant for monetary policy transmission. When credit costs are measured using the credit spreads, the credit channel's predominant transmission mechanism is via the commercial paper spread. Meanwhile, when credit costs are measured using the SLOOS variables, most of the

[77] See Table 3.1 in Section 3.2.

[78] This is particularly true for non-durable expenditures.

credit channel's transmission takes place via the credit demand variables. In light of these results, I study the composition of the credit channel next.

3.5.3 Decomposing the Credit Channel: Effect of Credit Costs Other Than Spreads

From the results in the previous section, it is clear that the credit channel is relevant for monetary policy transmission. What is still unclear is which ones are the relevant sub-channels within the credit channel. For instance, consider the case of non-durable expenditures. On one hand, when credit costs are measured purely with credit spreads, most of the transmission occurs via the commercial paper spread. On the other hand, when credit costs are measured just using SLOOS variables, most of the transmission happens via credit demand conditions. The observations from Sections 3.2 and 3.5.1 suggest that the information in the commercial paper spread is a subset of the information in the credit demand conditions. The question is then, how much of the transmission is *actually* happening via the commercial paper spread (part of the spread sub-channel) relative to other credit factors reflected by the credit demand conditions (part of the non-spread sub-channel)?

To answer this and other similar questions about the composition of the credit channel, I use the VAR_3 with the surprise change in the current fed funds future contract, FF_0, as the instrument.[79] I proceed in two steps. First, using the definitions $\bar{\gamma}_i$ and $\bar{\phi}_i^x$, I decompose the monetary policy transmission into three channels: conventional channel, spread sub-channel, and non-spread sub-channel.[80] I can then compare the contribution of each of these channels to gauge their importance. Second, I test for the statistical significance of the VAR coefficients associated with the spread and non-spread sub-channels. I do this to show that these two sub-channels are important not only in terms of their magnitude, but also in the sense that they are statistically significant.

Table 3.6 presents $\bar{\gamma}_i$ for forecast horizons of one, two and three years. In line with the conclusions from Section 3.5.2, a large portion (at least 60%) of the forecast error variance can be associated with specific transmission channels. Therefore, a decomposition of monetary policy into the three specific transmission channels is justified. To perform this decomposition, I first quantify the contribution of four components: price/rate (PR), spreads (SP), credit supply conditions other

[79] Two remarks about instrumenting the VAR_3 with the FF_3 instead. First, the results discussed here about the composition of the credit channel (contribution of spread and non-spread sub-channels) are unchanged. Second, the overall contribution of the credit channel becomes larger.

[80] The overall credit channel is composed of the spread and non-spread sub-channels.

Table 3.6: $\bar{\gamma}_i$ Under the VAR_3 Specification

	12 Month Horizon	24 Month Horizon	36 Month Horizon
1 YR Yield	93.8	85.1	80.4
CPI	66.9	67.2	64.4
Durables	84.6	79.7	78.5
Non-Durables	63.5	64.6	73.2
HH Debt	55.6	55.5	57.3

Note: Results are based on a one standard deviation of the monetary policy shock.

than spreads (PC_S), and credit demand conditions other than spreads (PC_D). The sets Γ_x, with $\text{x} \in \{\text{PR}, \text{SP}, \text{PC}_\text{S}, \text{PC}_\text{D}\}$, that identify each component are defined as in the previous section.[81] Table 3.7 presents the contribution of each of these four components to $\bar{\gamma}_i$. The first column gives the contribution of the Price/Rate channel. The second column refers to the transmission via the excess bond premium, the mortgage spread, and the commercial paper spread. The third and fourth columns refer to the contribution of the SLOOS credit supply and demand principal components, respectively. Both, PC_S and PC_D, exclude the "main" principal component of each block; hence they measure the transmission of policy via credit conditions orthogonal to the credit spreads. Note that the PC_S is part of the credit channel as it directly measures tightness of credit supply conditions. However, the PC_D indirectly measures the tightness of credit conditions via expansion/contraction of the demand. Therefore, it is a proxy of all factors that might affect credit demand, which potentially include both, the conventional channel and the non-spread credit channel. Given these components, I proxy the conventional channel with the PR component, the credit spread channel with the SP component, and the non-spread credit channel with the PC_S component. As I argue later, the PC_D components should also be included as part of the non-spread credit channel.

The results from Table 3.7 reaffirm the conclusions about the robustness of the conventional and credit channels. On one hand, the conventional channel is robust to the different measures of credit. As in both of the VAR_2 credit specifications, this channel's contribution is about $44\% - 52\%$ for households' debt, $17\% - 34\%$ for durable expenditures, and $37\% - 41\%$ for non-durable expenditures. On the other hand, the credit channel is not robust when it does not include the PC_D component. When only the SP and PC_S components are included, the credit channel's contribution is much smaller than under the VAR_2 specifications, being of only about $4\% - 5\%$ for households'

[81] The VAR_3 specification does not include the "main" principal components of the SLOOS credit supply and demand blocks, PC_S^1 and PC_D^2. Therefore, $\Gamma_{\text{SP}} = \{\text{EBP}, \text{Mort}_{\text{SP}}, \text{CP}_{\text{SP}}\}$ is uniquely defined.

Table 3.7: Contribution of Different Channels to the Monetary Policy Transmission Under the VAR_3

		Price/Rate	Spreads	PC_S	PC_D
12 Months	1 YR Yield	93.8	3.1	0.6	2.1
	CPI	64.1	5.9	6.8	11.0
	Durables	17.7	9.3	4.2	7.0
	Non-Durables	37.8	12.3	2.9	24.4
	HH Debt	44.8	1.9	2.3	6.2
24 Months	1 YR Yield	91.1	4.0	0.6	3.1
	CPI	55.4	11.1	7.9	11.4
	Durables	25.6	10.1	6.0	9.9
	Non-Durables	41.0	14.6	8.0	17.6
	HH Debt	48.3	2.3	2.7	9.2
36 Months	1 YR Yield	88.9	4.2	1.2	4.1
	CPI	55.5	13.3	8.1	9.7
	Durables	33.7	10.7	7.2	9.9
	Non-Durables	40.3	16.3	13.8	14.1
	HH Debt	51.5	2.4	3.0	9.1

Note: The contribution of each channel is measured by $\overline{\phi}_i^x$ and is based on a one standard deviation of the monetary policy shock.

debt, $13\% - 18\%$ for durable expenditures, and $15\% - 30\%$ for non-durable expenditures. However, if the PC_D component is added, the credit channel's contribution increases to $39\% - 44\%$ for non-durable expenditures and to $10\% - 14\%$ for households' debt, much like in the VAR_2 specifications.

Should the PC_D component be included as part of the credit channel? The answer depends on whether the indirect transmission mechanism captured by this component is mostly due to term-structure adjustments (conventional channel) or changes in non-spread credit factors (credit channel). Hence, to further understand this indirect transmission mechanism, consider two important observations from the VAR_2 analysis. First, when the SLOOS variables are excluded, this indirect mechanism is captured by the commercial paper spread. Second, when the SLOOS variables are included, this mechanism is mostly captured by the PC_D component. This suggest that this indirect mechanism is operating via a common factor related to both, the demand for credit and the commercial paper spread. Friedman and Kuttner (1993) provide some evidence about how changes in commercial paper spreads are related to the contraction/expansion in bank lending due to monetary policy actions. Note that this contraction/expansion in bank lending might also be reflected in the contraction/expansion of credit demand. If this is the case, the indirect transmission mechanism could be operating via bank lending conditions. Therefore, this evidence suggests including the PC_D component as part of the credit channel.[82]

Given these three components of the credit channel, I can assess the importance of the two credit sub-channels: spread and non-spread. The former consists of the SP component and it captures the transmission of policy via changes in the EBP, mortgage and commercial paper spreads. The later incorporates both, the PC_S and PC_D components and it captures the transmission via changes in credit conditions other than spreads. From Table 3.7, the non-spread sub-channel accounts for more than half of the transmission that takes place via the credit channel: roughly 83% for households' debt, about $63\% - 69\%$ for non-durable expenditures, and between $55\% - 61\%$ for durable expenditures. Overall, these results suggest that the role of the non-spread credit conditions is important for monetary policy transmission. Considering the overall size of the credit channel, this transmission mechanism (which is generally overlooked in the literature) is particularly important for non-durable expenditures.

So far, I have discussed the importance of the credit channel and its sub-channels for monetary policy transmission purely in terms of their size. In what follows, I briefly provide some evidence of the relevance of this channel and its components from a statistical point of view. In particular, I conduct some significance tests on the VAR_3 coefficients for the three credit spreads (EBP, Mort_SP, and CP_SP) and the SLOOS credit supply and demand principal components (PC_S^2, PC_S^3, PC_S^4, PC_D^1, and PC_D^3). Overall, these two sets of coefficient are jointly different from zero at the 1% significance level in each of the VAR_3 equations. I interpret this as evidence that both, spread and non-spread, sub-channels are statistically relevant within the credit channel.

Table 3.8 summarizes the results of the significance tests for each of the credit channel's components. In particular, the tests are based on an F-statistic under the null that the coefficients, for all lags, are jointly equal to zero. The columns represent the different credit channel components while the rows refer to selected equations of the reduced form VAR_3. For durable expenditures, all eight components are significant at the 1% level. For non-durables, only three of the components

[82]This indirect transmission mechanism is more significant for non-durable expenditures and households' debt. For durable expenditures, most of the transmission within the spreads credit channel takes place via the EBP and the mortgage spread rather than via the commercial paper spread.

Table 3.8: Robust F-statistic for the VAR_3 Coefficients (All Lags) of Each Component of the Credit Channel

	EBP	$Mort_{SP}$	CP_{SP}	PC_S^2	PC_S^3	PC_S^4	PC_D^1	PC_D^3
1 YR Yield	***	***	***	***	***	***	***	***
CPI	***	***		**	***	**	***	***
Durables	***	***	***	***	***	***	***	***
Non-Durables	*	**	***	**		**	***	***
HH Debt	***	***		***	***	**	***	***

Note: *** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

are significant at the 1% level: the commercial paper spread (CP_{SP}) and the credit demand components associated with mortgages (PC_D^1) and households' credit (PC_D^3). At the 5% significance level, three other components become significant; the mortgage spread ($Mort_{SP}$), the credit supply component associated with credit card loans (PC_S^2), and the credit supply component related to non-credit card loans (PC_S^4). Note that the excess bond premium becomes significant only at the 10% level. Interestingly, the component associated with credit supply conditions in the mortgage market (PC_S^3) is not statistically significant even at the 10% level. Finally, for households' debt, all of the components are significant at the 1% level except for two; the credit supply component associated with non-credit card loans (PC_S^4), and the commercial paper spread (CP_{SP}). The former becomes significant at the 5% level while the latter is not significant even at the 10% level.

The results from Table 3.8 are consistent with standard intuition. Given that durable expenditures are usually financed using one or more of these three types of loans (mortgage, credit card, and non-credit card), one can expect that the spreads, credit supply, and credit demand conditions in these three markets are all significant for these purchases. For non-durable expenditures, if they are mostly financed via credit card and non-credit card loans, the credit supply and demand conditions in these two credit markets ought to be significant. What is somewhat unexpected is that the credit conditions in the mortgage market (other than the spread) do not affect non-durable expenditures significantly. This could be an indication that mortgages are mostly used to finance the consumption of durable goods. Hence, the conditions in the mortgage market affect non-durable consumption only indirectly through their effect on the price of durable goods. As mortgages are used to finance durable consumption, the price of durables would reflect mortgage spreads but not other factors such as credit limits or credit score requirements. Note also that, although the credit supply conditions for mortgages are not significant, the credit demand conditions are. This is consistent with the interpretation for the credit demand conditions indirectly reflecting bank lending.[83] Regarding households' debt, note that it is defined as the sum of consumers' credit card, non-credit card and mortgage loans. Hence, it can easily be seen why the coefficients associated with the credit supply and demand conditions in the mortgage, credit card and non-credit card markets are all statistically significant.

In the end, the analysis of the VAR_3 specification yields two main results. First, the credit channel is relevant, both in terms of statistical significance and size, for the transmission of monetary policy to real activity. Although the credit channel is only about a fifth of the size of the conventional channel for households' debt, its contribution is much larger for durable and non-durable expenditures. It accounts for about a third of the transmission for the former and as much as half for the latter. Second, the composition of the credit channel is also important for monetary policy transmission. In addition to the standard transmission via credit spreads, the credit channel includes a non-spread component. This non-spread component roughly accounts for about 50% of the credit channel's total contribution for durable expenditures and as much as 70% for non-durable expenditures. Taken together, these results would suggest that in the study of monetary policy transmission, the credit channel can't be ignored. Furthermore, if this channel is introduced purely via financial frictions that are reflected as spreads, a large fraction of the transmission mechanism could be absent.

3.6 Conclusion

In this chapter, I quantify the contribution of three different channels for the transmission of monetary policy; the term structure of market interest rates (conventional channel), the spread over risk-free rates (credit spread channel), and the non-spread credit conditions (non-spread credit channel).

I am able to quantify the contribution of these channels using three key elements. First, I use data from the Senior Loan Officer Opinion Survey (SLOOS). As I show, this data contains more general information than the credit spreads of three important loan markets; commercial and industrial, mortgage, and commercial paper. I use this additional information to identify the non-spread credit

[83] However, there is also the possibility that the credit demand component, PC_D^1, reflects other factors such as income.

channel. Second, I use the external instrument identification approach proposed by Gertler and Karadi (2015). This scheme avoids making a priori restrictions on the interaction between the policy instrument and the other variables in the VAR model when identifying the policy shock. In turn, this allows me to quantify the contemporaneous effect (i.e. transmission) of unexpected policy announcements for each of the model variables. Lastly, I use a modification of the Forecast Error Variance Decomposition to be able to extend this measurement procedure to arbitrary forecast horizons.

I find that the credit channel (spread and non-spread) can account for as much as $20\% - 30\%$ of the variance of durable and non-durable expenditures following an unanticipated monetary policy shock. That is, the credit channel is as large as the conventional channel. Furthermore, corroborating the results of various recent empirical studies, I find that credit spreads are an important mechanism within the credit channel. Additionally, I expand this result by providing evidence that the adjustments in the non-spread factors play an important role too, specially for non-durable expenditures as they account for about 70% of the credit channel's total contribution.

Overall, my results suggest the importance of incorporating both, spread and non-spread credit channels in the study of monetary policy transmission. Including credit spread effects alone would leave a considerable fraction of the transmission mechanism absent, specially for non-durable purchases.

CHAPTER 4

Monetary Policy Revisited: Heterogeneous Bank Pass-Through of Credit Expansions

Abstract

This chapter implicitly accounts for heterogeneity in the pass-through of credit expansions from banks to households and assesses its impact on the monetary policy transmission mechanism to aggregate consumption. I build a model that embeds a financial friction in the Heterogeneous Agent New Keynesian (HANK) framework, which results in banks offering differentiated credit contracts to households in the economy. Following a credit expansion, the relaxation of credit conditions is about three times larger for households at the top of the wealth/income distribution relative to those at the bottom. This mechanism is able to replicate some recent empirical evidence showing that households with the highest credit ratings increase borrowing by twice as much relative to households with the lowest credit ratings after a change in banks' cost of funds. Incorporating this mechanism in the HANK framework reduces the response of aggregate consumption to a monetary expansion by about five times. This finding has potential implications for the distributional effects of monetary policy and for its control over aggregate responses.

JEL Codes: D14, E21, E43, E44, G10

4.1 Introduction

Two questions are central to the study of monetary policy: (i) does monetary policy affect aggregate outcomes? and, if it does, (ii) what are the relevant mechanisms by which it affects these outcomes? Although the affirmative answer to the first questions is relatively well establish and widely accepted, the quest for a satisfactory answer to the second question is still ongoing.[84] Recent studies have highlighted the role of heterogeneity, idiosyncratic uncertainty, and market incompleteness as key factors influencing the transmission of monetary policy.[85] Ultimately, idiosyncratic risk and market incompleteness lead to heterogeneity in households' marginal propensities to consume, which in turn is the driving force behind the consumption response. My intention is to complement this strand of literature by addressing another type of heterogenous effect present under idiosyncratic uncertainty and incomplete markets; heterogeneity in credit conditions across households. More importantly, as suggested by some empirical evidence, not only are credit conditions differentiated across households but the *change* in such conditions following a credit expansion is largely *non-homogeneous* across them.

This chapter explicitly models the pass-through of credit expansions/contractions from banks to households and assesses whether this mechanism is relevant in the transmission of monetary policy to aggregate outcomes. To this end, I build a model that captures market incompleteness, idiosyncratic income risk, financial frictions, and sticky prices. The novel feature is the introduction of debt negotiation between banks and households in the context of the Heterogeneous Agents New Keynesian (HANK) framework of Kaplan, Moll and Violante (2016). There is one asset in the economy which households can only trade via banks; thus banks can extract some surplus from households for offering the service. The credit conditions are negotiated between banks and households via Nash bargaining. Ultimately, the negotiated credit conditions are a function of the bank's and household's valuation of the service. The former is just a function of the amount of resources the bank can extract from the household while the latter is a function of the household's preferences, wealth, and income. The resulting mechanism is simple. Banks can extract more

[84] See for example the chapters of Christiano, Eichenbaum and Evans (1999) on the Handbook of Macroeconomics and of Boivin, Kiley and Mishkin (2011) on the Handbook of Monetary Economics. Both of these chapters provide evidence of the monetary policy effects on aggregate outcomes. The second chapter also provides a discussion about the different transmission mechanisms of monetary policy.

[85] See Luetticke (2015), Kaplan, Moll and Violante (2016), Auclert (2016) among others.

resources from richer households (higher wealth and income households); thus banks are willing to "relax" more the credit conditions for these households. Similarly, households who don't value the service as much require more "relaxed" credit conditions in order to agree to a contract. Under the usual assumptions about households' preferences (in particular concavity), households' valuation of the service is negatively correlated with wealth and income. Overall, the interaction of these two forces determines the pass-through of credit expansions/contractions.

As it is standard in the idiosyncratic income and incomplete markets literature, my model is able to capture several moments of the wealth and income distributions for the United States. In addition, the novelty of my model is that it is able to generate the distribution of changes in borrowing across households following a credit expansion as documented by Agarwal et al. (2016).

The study of Agarwal et al. (2016) uses credit card data for the U.S. over the period January 2008 - December 2014. They group households in different FICO score bins and quantify the average change in borrowing for each bin as the product of the Marginal Propensity to Borrow (MPB) and the Marginal Propensity to Lend (MPL). That is, the change in households' borrowing following a credit expansion depends on two factors. On one hand, the way in which banks adjust the credit card limits given the credit expansion; the MPL. On the other hand, the way in which households adjust their borrowing given the change in credit card limits; the MPB. Table 4.1 presents a summary of their results. First, given a $1 increase in credit limits, households on the highest FICO score bin increase their borrowing only by about two fifths relative to households on the lowest FICO score bin.[86] Second, following a credit expansion banks increase the credit limits by about five times more for the households in the highest FICO score bin relative to those in the lowest FICO score bin. The main takeaway is that there is a substantial degree of heterogeneity in the way in which households adjust their borrowing following a credit expansion. Furthermore, a non-trivial part of this heterogeneity stems from the way in which banks adjust the credit conditions for households. The end result is that following a credit expansions, households in the highest FICO score bin increase their borrowing by about two times more relative to households in the lowest FICO score bin.

[86]This is true for the debt measures that ignore portfolio effects; the Average Daily Balance and the Cumulative Purchase Volume. For the other two borrowing measures which capture portfolio effects, Interest Bearing Debt and Balances Across All Cards, households in the highest FICO score bin increase their borrowing by even a smaller amount relative to the lowest FICO score bin households.

Table 4.1: MPB, MPL, and MPB x MPL Ratios Given Different Debt Measures

	Avg. Daily Balance	Interest Bearing Debt	Balance All Cards	Cumulative Purchase Vol.
MPB	0.40	0.25	0	0.39
MPL	5.07	5.07	5.07	5.07
MPB x MPL	2.01	1.27	0	1.99

Note: The table is constructed using the results from Agarwal et al. (2016). The ratios are constructed by dividing the average for the households with the highest FICO scores (FICO > 740) by the average for the households with the lowest FICO scores (FICO ≤ 660).

My model is able to capture this empirical distribution of households' borrowing adjustments; it predicts households at the top of the model's FICO score distribution increase their borrowing by about 2.7 times more than the households at the bottom. The model's ability to replicate this empirical observation is a consequence of the mechanism introduced by the debt negotiation process; what I refer to as the "heterogenous bank pass-through". Given the model's calibration, there is a positive correlation between a household's model FICO score and her level of wealth and income. Given this correlation, the households with higher FICO scores are those who (i) have a relatively smaller valuation of a negotiation opportunity and (ii) have a larger amount of resources that can be extracted by a bank. These households would be willing to make adjustments in their borrowings only if large enough incentives (in the form of more relaxed credit conditions) are offered. Furthermore, the bank is willing to offer such incentives for these households given it can extract more resources from them. Thus, following a decrease in a bank's cost of funds, the bank relaxes the credit standards to all consumers but it does so more to those consumers with higher FICO scores. Overall, this implies that the adjustments in borrowing following a credit expansion are increasing in the FICO score, just as the empirical evidence suggests.

In the second part of the chapter, I use my model to assess the effect of the heterogeneous bank pass-through on the monetary policy transmission mechanism to aggregate outcomes. I show that when the heterogeneous bank pass-through mechanism is included within the standard HANK framework, the *direct* effect (i.e. ignoring general equilibrium effects) of a monetary policy expansion on aggregate consumption is about five times smaller upon impact. I obtain this result by comparing the response of consumption to a surprise change in the nominal interest rate under two scenarios; one without the financial friction (the standard HANK) and another one with the financial friction (HANK with the heterogeneous bank pass-through).

This result is mostly driven by the behavior of households with positive asset holdings. In the HANK framework, (i) consumption responds non-negatively for all households and (ii) the consumption response is larger for households with smaller wealth and income levels. When the heterogeneous bank pass-through is included only the latter is true. For households with sufficiently large wealth levels, consumption responds negatively to the decrease in the interest rate. In the standard HANK case, a decrease in the real rate maps one-to-one to a decrease in household's cost of credit. That is, the marginal propensity to lend to *all* households is one. When the heterogeneous bank pass-through is included, the marginal propensity to lend is (i) *less than one* to all households, and (ii) smaller to households with low wealth and income. Therefore, there is a much smaller substitution effect in the model with the heterogenous bank pass-through. Effectively, this renders the wealth effect of the decrease in the interest rate stronger, causing the decrease in consumption for wealthier households.

The dampening of the *direct* effect of a monetary expansion when the heterogenous bank pass-through is explicitly included has at least two relevant implications. The first one has to do with the size of the *indirect* general equilibrium effects of policy transmission. As some recent literature has concluded, the indirect effects are dominant under idiosyncratic uncertainty and incomplete markets. However, the indirect effects are just a propagation mechanism for the direct effects. Thus a dampening of the direct effects implies a smaller overall (direct plus indirect) effect of monetary expansions. The second implication is related to the degree of control that the monetary authority has over expansionary/contractionary policy. As emphasized in the literature that studies the indirect effects of policy, the responsiveness of aggregate consumption may be largely outside of the control of the monetary authority since the transmission takes place mostly via general equilibrium channels (profit redistribution, labor market adjustments, etc.). However, as my findings suggests, even for the direct effect on aggregate consumption the monetary authority must rely on the pass-through from banks to households.

The rest of the chapter is organized as follows. Section 4.2 discusses how this chapter fits within the literature. Section 4.3 presents the model and discusses in detail the outcome of the optimal contract. Section 4.4 analyzes the heterogenous bank pass-through mechanism arising from the contracting environment. Section 4.5 explains the calibration of the model. Section 4.6 presents the main results of the chapter. Finally Section 4.7 concludes.

4.2 Related Literature

In light of the previous results, this chapter contributes to the literature that incorporates market incompleteness and idiosyncratic uncertainty into New Keynesian models.[87] Relative to this literature, my paper is the first one to analyze monetary policy in a framework that explicitly models the pass-through of credit expansions/contractions from banks to households as documented in the empirical work by Agarwal et al. (2016).[88]

This chapter most closely relates to the work of Luetticke (2015) and Kaplan, Moll and Violante (2016). Both of these papers present DGSE models to study the transmission of monetary policy in a New Keynesian setting allowing for market incompleteness, idiosyncratic uncertainty, and a liquid-illiquid asset portfolio choice. The focus of these papers is to emphasize that the general equilibrium effects (the *indirect* effects) are far more important than the direct effect of monetary policy transmission. In contrast, this chapter focuses on the direct effect recognizing that (i) the indirect effect is just a propagation mechanism of the direct effect and (ii) the direct effect depends on how policy actions are transmitted from the banking sector to the households in the economy.

Relative to these papers, one short-coming of my paper is neglecting the household portfolio choice by focusing on the one asset case. This is important in at least two dimensions. First, this implies that I am unable to capture the effects of the wealthy hand-to-mouth, which are documented in Kaplan and Violante (2014). However, the wealthy hand-to-mouth consumers play an important role for the indirect effects of policy transmission, which is not the main focus of this chapter. Second, the portfolio choice might allow wealthy households to shield themselves from the wealth effect arising from a decrease of the real risk free rate. However, even if households hold a portfolio which shields them from the wealth effect, the additional credit could result in a portfolio rebalancing instead of an increase in consumption; thus dampening the response of aggregate consumption. As a matter of fact, empirical studies such as Agarwal et al. (2016) and Díaz-Giménez, Glover and

[87] Guerrieri and Lorenzoni (2011), Oh and Reis (2012), Ravn and Sterk (2014), Haan, Rendahl and Riegler (2015), Bayer et al. (2015), McKay and Reis (2016), McKay, Nakamura and Steinsson (2016), Auclert (2016), Gornemann, Kuester and Nakajima (2016), Werning (2015), Luetticke (2015), Kaplan, Moll and Violante (2016), among others.

[88] At least to the best of my knowledge. Note that the pass-through of credit (financing) expansions/contractions has been extensively studied on the firm side. In the early 90's, a strand of literature started examining the role of credit market frictions for monetary policy transmission via firms' borrowing. This literature is best exemplified by papers such as Bernanke (1993), Bernanke and Gertler (1995) and the seminal contributions of Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) which provided a micro-founded link between credit market frictions and the resulting credit costs.

Ríos-Rull (2011), have documented that households indeed rebalance their portfolio in response to an interest rate cut. For instance, in Table 4.1, when borrowing is measured using variables that incorporate portfolio effects (i.e. Balance Across All Cards), the households with the highest FICO scores do not increase credit card borrowing at all in response to a credit expansion.[89]

Additionally, my paper expands on the existing line of work that evaluates whether empirical micro findings about households' consumption responses to policy are relevant from a macro perspective.[90] In particular, I complement the empirical evidence provided by Agarwal et al. (2016). They suggest that the negative correlation between a bank's marginal propensity to lend and households' marginal propensity to borrow might be important for the transmission of credit expansions to aggregate consumption. I elaborate on this suggestion and examine it in a DSGE framework.

4.3 The Model

Time is continuous and there are no aggregate shocks. I consider a partial equilibrium economy as it suffices for the study of the mechanism I have in mind.[91] The economy is populated by three types of agents: households, banks, and the government; all of whom are infinitely lived. Households are risk averse and consume a non-durable good, supply labor, face idiosyncratic income risk, and have access to a private real asset b which can be used as either a borrowing or saving instrument. I assume there is a trade friction on the private asset market in the form of infrequent renegotiation of the asset position.[92] In particular, renegotiation happens at an exogenous rate σ. Once a bank and a household meet for renegotiation, they contract on a new asset position b' and a corresponding

[89] I am currently working on a version of my model allowing for liquid and illiquid assets. My goal is to (i) quantify the effect of the heterogeneous bank pass through on the indirect effects of policy and (ii) be able to match the second finding of Agarwal et al. (2016) which is that households with the highest FICO scores have the lowest marginal propensities to borrow.

[90] For instance Johnson, Parker and Souleles (2006), Parker et al. (2013), or Misra and Surico (2014), among others, document the response of consumption to transfers.

[91] The model can be closed using the Heterogeneous Agent New Keynesian framework (HANK) as in Kaplan, Moll and Violante (2016). I am currently working in this general equilibrium version of the model to explore the relevance of the heterogeneous bank pass-through channel compared to other transmission channels that arise due to agent heterogeneity.

[92] On the borrowing side, this assumption is motivated by the observation that for certain types of borrowing instruments (for instance credit cards), banks periodically send "upgrade" offers notifying customers they qualify for better credit standards (i.e. increase in credit limits). On the savings side, this assumption is motivated by the observation that, for certain types of assets, the transactions are done via a dealer. Therefore, it might be costly (in terms of search time) to find one. Alternatively, one can interpret this assumption as capturing savings instruments (such as savings accounts) which may require minimum balances, minimum deposit amounts, limits on the number of transactions, or fixed deposit terms. A household who might have agreed to such conditions might violate them on occasion due to unforeseen circumstances (such as accidents or illness). In this sense, the asset in the model is not a purely liquid asset and should be seen more as representing a mixture of assets of different degrees of liquidity.

one time interest rate payment $\tilde{r}$ (which can be equivalently seen as a transaction fee ϕ) via Nash bargaining. Banks are risk neutral and they have access to two types of assets; the private asset b and government debt B_g. The latter is traded in a perfectly competitive market. In essence, banks just provide financial intermediation between private agents and between the private agents and the government. The government finances some exogenous spending G by collecting taxes/transfers τ from the agents in addition to issuing debt.

4.3.1 Households

The economy is populated by a continuum of households with measure one who are heterogeneous in two dimensions: their asset holdings b, and their idiosyncratic labor productivity z. While the former is a consequence of endogenous choices of the agents, the latter is assumed to be an exogenous Markov chain, which I describe in detail in Section 4.5.1.

At each instant t, households receive a utility flow $u(\cdot)$ from consumption c_t and work l_t. The utility function is assumed to be strictly increasing and strictly concave in consumption and strictly decreasing and strictly convex in work. Households are assumed to have time-separable preferences and to be impatient with a discount rate of $\rho \geq 0$. Hence the lifetime utility for a household is given by

$$U = \mathbb{E}\int_0^\infty e^{-\rho t}u(c_t, l_t)dt, \tag{4.3.1}$$

where the expectation is taken over the realizations of the idiosyncratic productivity process and the renegotiation offers. Note that due to the law of large numbers there is no aggregate uncertainty and the state of the economy is completely captured by the joint distribution $g_t(db, dz)$.

When making their choices of the consumption and hours worked, households take as given the paths of the government's function for taxes/transfers $\{\tau_t(\cdot)\}_{t\geq 0}$, real wage rate $\{w_t\}_{t\geq 0}$, and the net real return on government debt $\{r_t\}_{t\geq 0}$.[93] At any instant t, households are uncertain of whether or not they are going to be able to renegotiate their asset holdings with their bank. The

[93] In the HANK framework the paths of all these prices would be determined as an equilibrium outcome. In my formulation, I am interested in isolating the transmission of monetary policy via the changes in borrowing/lending associated with a change in the bank's cost of funds (in my set up, the net return on government debt). Therefore, as in Kaplan, Moll and Violante (2016), to isolate such effect from the general equilibrium feedback effects via taxes and wages, I keep those prices fixed at their steady state values. This is the sense in which for my purposes, a partial equilibrium framework suffices.

renegotiation opportunities follow an exogenous Poisson process with rate σ. Therefore, households make their consumption and hours worked choices contingent on renegotiation $\left\{c_t^{\mathrm{R}}, l_t^{\mathrm{R}}\right\}_{t\geq 0}$ and no renegotiation $\left\{c_t^{\mathrm{NR}}, l_t^{\mathrm{NR}}\right\}_{t\geq 0}$. In addition, conditional on renegotiation, they must agree with their bank on the new level of asset holdings and adjustment fee $\{b_t', \phi_t\}_{t\geq 0}$. The contracting environment that determines the resulting (b_t', ϕ_t) is described in detail in Section 4.3.3.

Under this set up, a household's net savings evolve according to

$$dS_t = \begin{cases} b_t' - b_t + \phi_t, & \text{if renegotiation} \\ 0, & \text{otherwise,} \end{cases}$$

where it is assumed that to be able to renegotiate the asset position the household must first repay/collect the existing obligations/deposits b_t. The budget constraint implies that the household's net savings dS_t must equal the household's income stream (labor income plus asset interest income) net of government taxes and consumption:

$$\begin{aligned} b_t' - b_t + \phi_t &= w_t z_t l_t^{\mathrm{R}} + r_t b_t - c_t^{\mathrm{R}} - \tau_t\left(w_t z_t l_t^{\mathrm{R}}\right), \quad \text{if renegotiation.} \\ 0 &= w_t z_t l_t^{\mathrm{NR}} + r_t b_t - c_t^{\mathrm{NR}} - \tau_t\left(w_t z_t l_t^{\mathrm{NR}}\right), \text{ otherwise.} \end{aligned} \tag{4.3.2}$$

Therefore, the household's problem is to choose the paths $\left\{c_t^{\mathrm{R}}, l_t^{\mathrm{R}}, c_t^{\mathrm{NR}}, l_t^{\mathrm{NR}}\right\}_{t\geq 0}$ to maximize (4.3.1) subject to (4.3.2) taking as given the paths for prices and the path of renegotiated contract outcomes. The recursive formulation of the household's problem is presented in Section 4.3.4.

4.3.2 Banking Sector

The banking sector consists of a main bank and a continuum of affiliated bank branches of measure one. Each of these branches is tied to a specific consumer in the economy and acts as a financial intermediary between the household and the main bank.

At each instant t, the branch simply transfers the interest payment $r_t b_t$ from the main bank to its corresponding household (and vice versa). If a renegotiation opportunity arises, in addition to transferring resources from the main bank to the household, the branch is able to extract some surplus from the transaction.[94] I assume the branches are risk neutral agents who instantaneously consume

[94]The branch has some monopolistic power since I assume this is the only vehicle by which the household can get in

all the surplus they extract. Conditional on renegotiation, (i) a household with asset holdings b_t and productivity level z_t repays the existing balance b_t and requests an amount $b'_t(b_t, z_t)$; (ii) the branch charges the household an associated instantaneous interest rate $\tilde{r}_t(b_t, z_t)$; (ii) the branch requests the amount $b'_t(b_t, z_t)$ from the main bank and transfers b_t; (iii) the main bank gives the branch $b'_t(b_t, z_t)$ to transfer to the household and in turn requires compensation equal to the net real return r_t (the opportunity cost of the funds).

It must be noted that there are some implicit assumptions in the previous formulation. First, the bank branch has perfect monitoring technology so that both, b' and ϕ, can be made contingent on the households asset holdings b_t and productivity level z_t. Second, the interbank market between the branch and the main bank is perfectly competitive. Third, the main bank is also a risk neutral agent. Lastly, the main bank has access to government debt. This last three assumptions imply that the main bank charges the branch an amount equal to r_t, which is the opportunity cost of the funds.[95]

Given this setup, the consumption c_t^B of a branch that is tied to an agent with asset holdings b_t and labor productivity z_t is given by:

$$\begin{aligned} c_t^{\text{B,R}} &= \big(r_t - \tilde{r}_t(b_t, z_t)\big) \cdot b'_t(b_t, z_t), \quad && \text{if renegotiation} \\ c_t^{\text{B,NR}} &= 0, && \text{otherwise.} \end{aligned}$$

Due to the continuous time nature of the problem, if a branch in this economy is tied to a household with holdings b_t, the branch must make an instantaneous interest payment of $r_t \cdot b_t$. However, if the household instantaneously adjusts her asset holdings from b_t to b'_t, the instantaneous interest payment the branch must make is now $\tilde{r}_t \cdot b'_t$. With this in mind, define $\phi_t \equiv r_t b_t - \tilde{r}_t b'_t$; it is as if the branch's interest payment to the household is $r_t \cdot b_t$ at every instant in time (whether the assets are adjusted or not) but the branch charges and additional fee of ϕ_t when there is renegotiation that leads to an adjustment in the asset holdings.

Similarly, the branch must get the interest payment to transfer to the household from the main bank. For a household with asset holdings b_t, the branch must get $r_t \cdot b_t$ from the main bank. However, if the household adjusts her position, the branch must now get $r_t \cdot b'_t$. Again, it is as if

touch with the main bank. In my model, the "main bank" is a very crude representation of the interbank lending market. Additionally, I assume household's can't borrow/lend directly from each other but must do so via the banking sector.

[95]The main bank can have access to some other type of assets whose return is linked to r_t by a no arbitrage argument. What is important for my formulation is that the cost of funds is a function of r_t.

the branch is always getting the funds $r_t \cdot b_t$ from the main bank to transfer to the household and whenever there is renegotiation that leads to an adjustment in the asset holdings the main bank just charges the branch a fee of $r_t \cdot b_t - r_t \cdot b_t'$

With this alternative interpretation, the branch's consumption is given by

$$\begin{aligned} c_t^{\text{B,R}} &= \phi(b_t, z_t) - r_t\big(b_t - b_t'(b_t, z_t)\big), && \text{if renegotiation} \\ c_t^{\text{B,NR}} &= 0, && \text{otherwise.} \end{aligned} \tag{4.3.3}$$

That is, the branch behaves as if it would charge an instantaneous fee ϕ_t to the consumer for the renegotiation procedure while paying the main bank the net interest it could have made on those funds (the cost of funds). I prefer this interpretation because it emphasizes that the transaction between the branch and the consumer is an instantaneous transaction and doesn't have any intertemporal effects from the branch's perspective. Negotiating a one time fee conveys this idea better than negotiating on an interest rate, which is usually related to intertemporal payments.

4.3.3 Contract

When a renegotiation opportunity arises at instant t, a branch and a household with asset holdings b and productivity level z contract on (i) the adjusted level of asset holdings b' and (ii) the instantaneous fee paid to the branch ϕ.[96] Given that the branch has perfect monitoring technology, b' and ϕ can be contracted contingent on the household's type (b, z).[97]

The contracting happens via Nash bargaining, where θ denotes the branch's bargaining power and $1 - \theta$ the household's bargaining power. I assume that there is perfect commitment from the household's side. Therefore, the no agreement outcome is simply the same as if there was no renegotiation: the household keeps her current asset holdings $b' = b$ and hence pays no fee (i.e. $\phi = 0$).

Under this set up, for a household of type (b, z) who requests funds b' and pays ϕ, the surplus from the contract is given by:

[96]Completely equivalent results can be derived if instead the contract is on the adjusted level of asset holdings b' and the instantaneous one time interest rate $\tilde{r}$.

[97]Strictly speaking, b' and ϕ are contracted contingent not only on the household's type but also on the aggregate state of the economy Ψ_t.

$$CS_t = \left[V_t(b', z; \Psi_t) + u(c_t^{\mathrm{R}}, l_t^{\mathrm{R}})\right] - \left[V_t(b, z; \Psi_t) + u(c_t^{\mathrm{NR}}, l_t^{\mathrm{NR}})\right], \tag{4.3.4}$$

where $c_t^{\mathrm{R}} = c^{\mathrm{R}}(b, z; \Psi_t, b', \phi)$, $l_t^{\mathrm{R}} = l^{\mathrm{R}}(b, z; \Psi_t, b', \phi)$, $c_t^{\mathrm{NR}} = c^{\mathrm{NR}}(b, z; \Psi_t)$, and $l_t^{\mathrm{NR}} = l^{\mathrm{NR}}(b, z; \Psi_t)$ are the optimal policy functions associated with the value function $V_t(b, z; \Psi_t)$.[98] Note that $\Psi_t = \left\{r_q, w_q, \tau_q(\cdot)\right\}_{q \geq t}$ is an object that captures the aggregate state of the economy, which is given by the path of prices and the government tax function. In what follows, and for notational convenience only, I suppress the explicit dependence of the value function and associated optimal policies on Ψ_t.

The first term in brackets in (4.3.4) is the household's total utility when an agreement is reached: the household enjoys the continuation value given the adjusted asset holdings $V_t(b', z)$ and the utility flow derived from her instantaneous consumption $c_t^{\mathrm{R}}(b, z; b', \phi)$ and labor $l_t^{\mathrm{R}}(b, z; b', \phi)$. The second term in brackets captures the household's total utility without agreement: the continuation value is just $V_t(b, z)$ and the current utility flow is derived from the no renegotiation consumption $c_t^{\mathrm{NR}}(b, z)$ and labor $l_t^{\mathrm{NR}}(b, z)$.

Note that if an agreement is reached, the only adjustments are the asset holdings b' and the fee ϕ. In particular, the household's productivity level remains fixed. This is a consequence of the continuous time formulation of the problem. In particular, modeling the renegotiation offer and the income productivity processes as continuous-time Markov chains implies that for any infinitesimal time interval only one of two events can occur: a renegotiation offer is extended or a productivity shock occurs.

As for the branch, its surplus from the contract when extending funds b' and charging a fee ϕ to a household of type (b, z) is given by

$$BS_t = \phi + r_t\left(b' - b\right), \tag{4.3.5}$$

which is straightforward and follows from (4.3.3). The resulting Nash bargaining problem is then

$$\begin{aligned} &\max_{b', \phi}\ CS_t^{1-\theta} \cdot BS_t^{\theta} \\ &s.t \\ &\quad CS_t, BS_t \geq 0 \\ &\quad b' > \underline{\mathrm{b}} \end{aligned} \tag{4.3.6}$$

That is, the optimal asset adjustment quantity $b'(b, z)$ and fee charged by the bank $\phi(b, z)$ maximize

[98] See Section 4.3.4 for the recursive formulation of the household's problem.

a weighted average of the household's and branch's surplus, subject to the constraint that both parties must actually benefit from the transaction. Additionally, I assume that there is an exogenous borrowing limit $\underline{b}$.

4.3.4 Household's Value Function and Optimal Policies

Let $\Psi_t = \left\{ r_q, w_q, \tau_q(\cdot) \right\}_{q \geq t}$ capture the aggregate state of the economy. Consider a household with asset holdings b and idiosyncratic income productivity $z = e^{z^P + z^T}$ (where z^P and z^T refer to the permanent and transitory components). Furthermore, let b' and ϕ denote the outcome of the contract if the household gets a renegotiation offer from her bank.[99] Then the problem can be formulated recursively as:

$$
\begin{aligned}
\rho V_t(b, z^P, z^T; \Psi_t) = & \max_{\{c^{\mathrm{R}}, l^{\mathrm{R}}, c^{\mathrm{NR}}, l^{\mathrm{NR}}\}} u(c^{\mathrm{NR}}, l^{\mathrm{NR}}) + \sigma \left[u(c^{\mathrm{R}}, l^{\mathrm{R}}) - u(c^{\mathrm{NR}}, l^{\mathrm{NR}}) \right] \\
& + \sigma \left[V_t(b', z^P, z^T; \Psi_t) - V_t(b, z^P, z^T; \Psi_t) \right] \\
& + \sum_{\tilde{z}^P \neq z^P} \lambda^P(z^P, \tilde{z}^P) \left[V_t(b, \tilde{z}^P, z^T; \Psi_t) - V_t(b, z^P, z^T; \Psi_t) \right] \\
& + \sum_{\tilde{z}^T \neq z^T} \lambda^T(z^T, \tilde{z}^T) \left[V_t(b, z^P, \tilde{z}^T; \Psi_t) - V_t(b, z^P, z^T; \Psi_t) \right] \qquad (4.3.7) \\
& + \partial_t V_t(b, z^P, z^T; \Psi_t)
\end{aligned}
$$

s.t.

$$
\begin{aligned}
c^{\mathrm{R}} &= w_t z l^{\mathrm{R}} - \tau_t \left(w_t z l^{\mathrm{R}} \right) + r_t b - (b' - b) - \phi \\
c^{\mathrm{NR}} &= w_t z l^{\mathrm{NR}} - \tau_t \left(w_t z l^{\mathrm{NR}} \right) + r_t b
\end{aligned}
$$

This recursive formulation can be derived from the discrete time version of the problem with time periods of length Δ and then taking the limit $\Delta \to 0$.[100] Intuitively, the total value flow for the household at any instant t is derived from three sources. First is the utility flow associated with instantaneous consumption and labor $u(c, l)$. Within an infinitesimal instant dt, the household gets a renegotiation offer with probability σdt so consumption and labor are c^{R}, l^{R} rather than c^{NR}, l^{NR}. Second is the continuation value, which reflects a change in the asset position or productivity level of the household. Since the renegotiation offer and the income productivity processes are modeled

[99] Note that in choosing her optimal consumption and labor, the household takes the outcome of the contract as given.

[100] The derivation is an application of the procedure presented in Appendix A of Achdou et al. (2015).

as continuous-time independent Markov chains, with probability σdt the household gets a renegotiation offer; with probability $\lambda^P(z^P, \tilde{z}^P)dt$ the households's permanent productivity component goes from z^P to $\tilde{z}^P$; and with probability $\lambda^T(z^T, \tilde{z}^T)dt$ the households's transitory productivity component goes from z^T to $\tilde{z}^T$. However, for any infinitesimal instant dt, at most one of this three changes can happen. Finally, is the change in value associated with the change in the aggregate state.

Note that the stationary version of the problem (associated with the invariant distribution in the economy) is obtained for the case $\Psi_t = \overline{\Psi}$. This implies that $r_t = \bar{r}$, $w_t = \bar{w}$, and $\tau_t(\cdot) = \bar{\tau}(\cdot)$ are fixed at their steady state levels and $\partial_t V_t(b, z^P, z^T; \Psi_t) = 0$.

The following proposition characterizes the policy functions associated with the value function in (4.3.7).

Proposition 4.1 *Consider a household with asset holdings b, productivity level z, and contract offer* (b', ϕ). *Assume:*

1. $u(c,l) = \dfrac{[c - g(l)]^{1-\gamma} - 1}{1-\gamma}$, *with* $\gamma > 1$
2. $g(l) = \dfrac{\psi z l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$
3. $\tau_t(x) = \tau_t^l \cdot x + T_t$

Define $y_t \equiv \left(1 - \tau_t^l\right) w_t z l^* + r_t b + T_t$ *and* $l_t^* \equiv \left(\dfrac{\left(1-\tau_t^l\right) w_t}{\psi}\right)^\nu$, *then the optimal policy functions satisfy*

(a) $l_t^R(b, z; \Psi_t, b', \phi) = l_t^{NR}(b, z; \Psi_t) = l_t^*$

(b) $c_t^{NR}(b, z; \Psi_t) = y_t$

(c) $c_t^R(b, z; \Psi_t, b', \phi) = y_t - (b' - b) - \phi$

The third assumption is just stating that the tax function is composed of a proportional labor income tax τ_l and a lump sum transfer T.[101] The first and second assumptions state that the instantaneous utility over consumption and labor is as in Greenwood, Hercowitz and Huffman (1988),

[101] I choose this structure of the tax function so that I can match in the calibration the percent of the population getting a net transfer. This is not of fundamental importance for the transmission of policy via the direct effect but does play a role for the indirect effet.

with the slight modification that the labor disutility depends also on the productivity level z. This implies that the labor supply responds only to the changes in the effective (net of tax) aggregate wage rate. Consequently, the labor supply function is independent of whether the household gets a renegotiation offer or not.

When the household doesn't get a renegotiation offer, her consumption c_t^{NR} is just her total current income (labor plus interest on her asset holdings). If the renegotiation offer is extended, the household's consumption c_t^{R} is her total current income adjusted by her change in the asset position $b' - b$ and the fee payment amount ϕ. Finally, given that the instantaneous utility is strictly increasing and concave, the value function that solves (4.3.7) is a monotone non-decreasing and concave function of b.[102]

4.3.5 Contract Solution

The optimal policy functions specify the household's behavior given (i) the aggregate state of the economy Ψ_t and (ii) the outcome of the contract (b', ϕ). For notational convenience, in what follows I denote the optimal labor supply function as l_t^* and the consumption policy function conditional on no renegotiation simply as c_t^{NR}. Then from Proposition 4.1, the consumption policy function conditional on renegotiation is $c_t^{\mathrm{R}} = c_t^{\mathrm{NR}} - (b' - b) - \phi$.

Proposition 4.2 *Consider a household with asset holdings b and productivity level z. In addition to the assumptions in Proposition 4.1 assume:*

1. $-\left(\dfrac{\theta}{1-\theta}\right)\left(\dfrac{1}{1-\gamma}\right) = 1$

Then

(a) The set of feasible renegotiated asset holdings b' is given by $\left(\underline{b}, \bar{b}.\right)$ for some $\underline{b} \leq b \leq \bar{b}$.

(b) The fee associated with each feasible level of asset holdings is given by

$$\phi(b') = \left[c_t^{NR} - (b' - b) - g(l^*)\right] - \left(\frac{c_t^{NR} - (1 - r_t)(b' - b) - g(l_t^*)}{\frac{\theta}{1-\theta}\left(V_t(b', z) - V_t(b, z) - (1-\gamma)^{-1}\left[c_t^{NR} - g(l_t^*)\right]^{1-\gamma}\right)}\right)^{1/\gamma} \tag{4.3.8}$$

[102] I don't have a formal proof of this statement. Intuitive (yet informal) arguments would suggest it is true, as well as all the numerical experiments I have performed.

The assumption is a normalization assumption. Effectively, it is controlling the branch's bargaining power and hence the extent to which the bank can extract surplus from the household via ϕ.[103] If both agents were risk neutral, the total utility of the match associated with the current period utility flow would be given by:

$$TS_t = \underbrace{r_t(b' - b) + \phi}_{\text{bank}} + \underbrace{c_t^{\text{NR}} - (b' -' b) - \phi - g(l_t^*)}_{\text{household}} = c_t^{\text{NR}} - (1 - r_t)(b' - b) - g(l_t^*).$$

That is, ϕ would be a transfer of funds from one agent to the other. Thus the assumption in Proposition 4.2 ensures ϕ reflects only a transfer of funds between agents when one of them is risk averse.

The first implication of Proposition 4.2 is that the feasible borrowing/lending amount is bounded above and below. Given the exogenous borrowing constraint, the lower bound is obvious. However, even in the absence of such exogenous limit, there is a lower bound on the borrowing amount. This follows from the monotonicity of the value function and the Inada type conditions that the instantaneous utility satisfies. Intuitively, as b' becomes small (very large borrowing), the household's current utility flow gets close to its upper bound ($\lim_{c\to\infty} u(c, l^*) = \gamma - 1$) while the continuation value $V_t(b', z)$ keeps decreasing. At some point $\underline{\text{b}}$ the trade-off is too large and the household does not benefit from additional borrowing. The existence of the upper bound follows a similar logic. The instantaneous utility as in Greenwood, Hercowitz and Huffman (1988) implies that there is a minimum (subsistence) consumption level, $\underline{\text{c}}_t = g(l_t^*)$. As b becomes large (very large savings), for some $\bar{b}$ the household eventually reaches $\underline{\text{c}}$ and hence, not matter how large the continuation value $V_t(b', z)$ is, she finds it unattractive to increase her assets any further. That $b \in (\underline{\text{b}}, \bar{b})$ follows simply from the fact that it is always feasible for the household to keep her current level of asset holdings and pay no fee.

The second implication of Proposition 4.2 is about how the branch sets the menu $\phi(b')$. One can think of the contract in the following way. For any feasible choice of b', the household is willing to give the branch at most a fraction $\frac{\theta}{1-\theta}$ of the surplus (CS_t) she gets. This implies the bank can extract at most $\frac{\theta}{1-\theta}\, \text{CS}_t\, u_c^{-1}$ current consumption units using the fee (ϕ); this consumption amount evaluated at the margin would yield a utility value exactly equal to the surplus the household is willing to give up. Therefore, the branch would set ϕ so as to extract this largest possible current

[103]For $\gamma \in (1, 2]$ the corresponding bank's bargaining power θ is in the range $(0, 0.5]$.

consumption units from the household.

Equation (4.3.8) provides a mathematical expression for the intuition provided in the previous paragraph. To see this, note that the household's surplus is completely independent of ϕ except for the current utility flow. With this in mind, define

$$\widetilde{CS}_t(b, z, b'; \Psi_t) \equiv V_t(b', z) - V_t(b, z) - u(c_t^{\text{NR}}, l^*) - (1 - \gamma)^{-1}, \tag{4.3.9}$$

$$\widetilde{BS}_t(b, z, b', \phi; \Psi_t) \equiv r_t(b' - b) + \phi - \left(\frac{\theta}{1 - \theta}\right) \frac{u(c_t^{\text{R}}, l_t^*)}{u_c(c_t^{\text{R}}, l_t^*)}. \tag{4.3.10}$$

$\widetilde{CS}_t$ is the portion of the household's surplus that is independent of ϕ while $\widetilde{BS}_t$ is the effective units of consumption the branch extracts from the household (note the effect of ϕ on the current utility flow - the last term). With these adjustments, it is as if the household promises to give the branch the fraction $\frac{\theta}{1-\theta}$ of her *effective* surplus $\widetilde{CS}_t$ while the branch *effectively* extracts $\widetilde{BS}_t$ units from her. Hence the branch sets ϕ so that the marginal value of the effective consumption units extracted from the household exactly equals the promised fraction of her effective surplus:

$$\left(\frac{\theta}{1 - \theta}\right) \widetilde{CS}_t = \widetilde{BS}_t \cdot u_c(c_t^{\text{R}}, l_t^*). \tag{4.3.11}$$

Note that another interpretation of this expression is that, at the margin, the branch's valuation of the service is the same as that of the household.

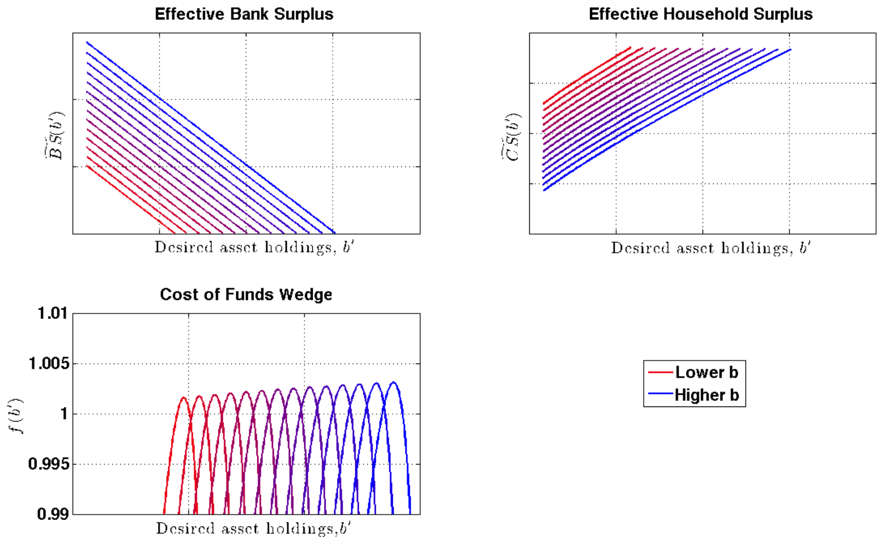
The expression in (4.3.8) incorporates one further simplification. In light of the previous discussion, it can be seen that ϕ has two important effects. First, it directly affects the effective consumption units the branch extracts from the household, $\widetilde{BS}_t$. Second, it affects the marginal value of those units, $u_c(c_t^{\text{R}}, l_t^*)$. The assumption in Proposition 4.2 eliminate the effect of ϕ on the amount of consumption the branch can extract. In particular, with this assumption,

$$\widetilde{BS}_t(b, z, b'; \Psi_t) = c_t^{\text{NR}} - (1 - r_t)(b' - b) - g(l_t^*), \tag{4.3.12}$$

is fixed and independent of ϕ.

Evidently, the effective household ($\widetilde{CS}_t$) and branch ($\widetilde{BS}_t$) surpluses play an important role in determining the fee set by the branch. Furthermore, as I will shortly show, the financial friction

Figure 4.1: Comparative Statics With Respect to b'



introduces a household specific wedge in the cost of funds, $f(b, z, b'; \Psi_t)$; which is directly related to these two surpluses. With this in mind, the following proposition characterizes the comparative statics of the effective household and branch surpluses.

Proposition 4.3 *Let b denote the household's asset holdings, z the household's productivity, and b' the requested amount of funds. Given the assumptions of Propositions 4.1 and 4.2, then*

1. $\dfrac{\partial \widetilde{CS}_t}{\partial b'}, \dfrac{\partial \widetilde{BS}_t}{\partial b}, \dfrac{\partial \widetilde{BS}_t}{\partial z} \geq 0.$

2. $\dfrac{\partial \widetilde{BS}_t}{\partial b'}, \dfrac{\partial \widetilde{CS}_t}{\partial b}, \dfrac{\partial \widetilde{CS}_t}{\partial z} \leq 0.$

Although I do not provide a formal proof of this proposition, I present a numerical characterization of these comparative statics exercises in Figures 4.1 - 4.3.

As shown in the top panels of Figure 4.1, the effective branch surplus is decreasing in b' (left panel) while the household surplus is increasing (right panel). The graphs correspond to a fixed productivity level $\bar{z}$ for various levels of current wealth b. As b' increases, the branch has a smaller incentive to extract additional resources from the household using the fee since it already gets resources in the form of deposits. For the household, increasing her deposits increases her valuation of the service since her continuation value function is non-decreasing in the level of asset holdings.

Figure 4.2: Comparative Statics With Respect to b

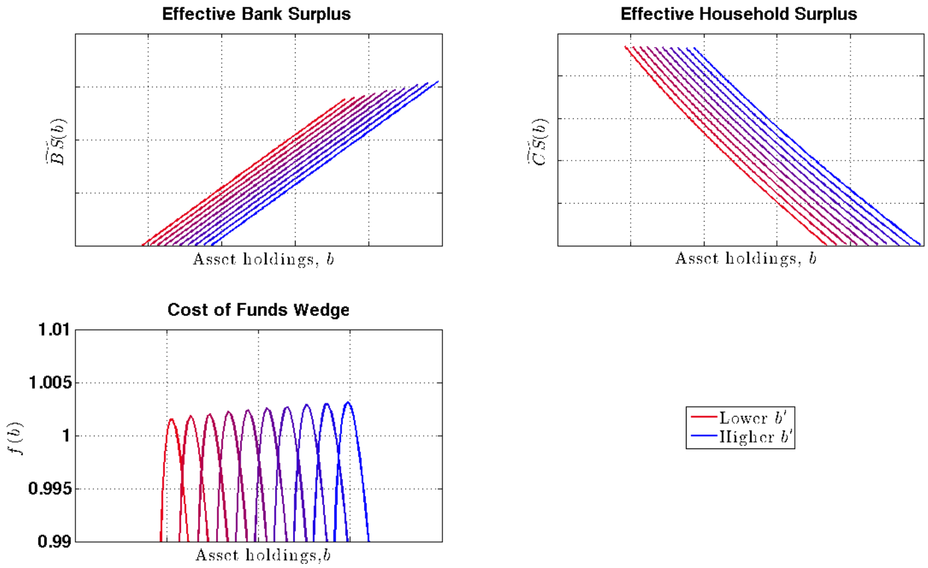
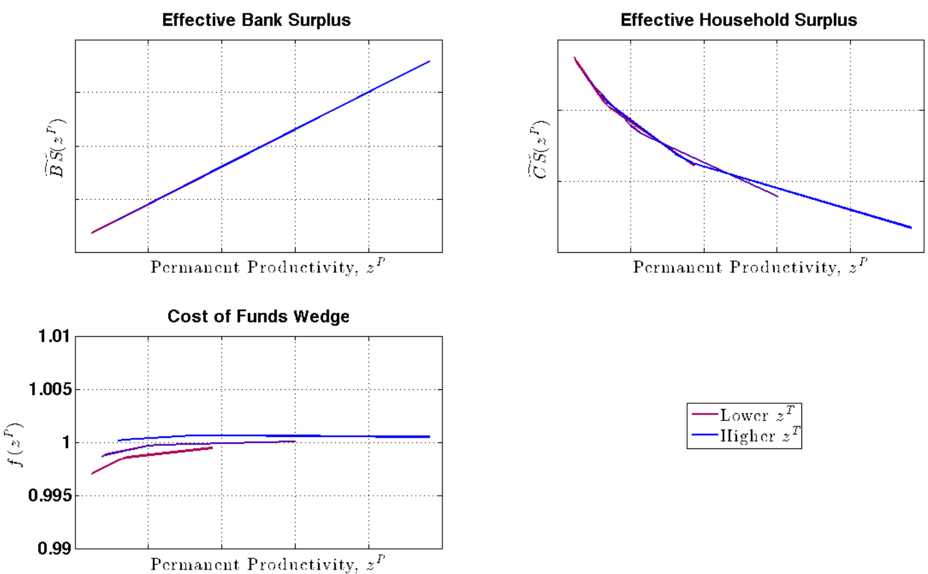


Figure 4.3: Comparative Statics With Respect to z



The top panels of Figure 4.2 show, for a fixed productivity level $\bar{z}$ and different desired asset positions b', that the effective branch surplus is increasing in b (left panel) while the household surplus is decreasing (right panel). As her wealth increases, the household's value for the service decreases since now she is able to have both, a larger continuation value and a larger current consumption, without readjusting her asset position. For the branch, meeting a wealthier household implies it can extract more resources via the fee regardless of her desired level of deposits/borrowing.

Lastly, Figure 4.3 shows that the effective branch surplus is increasing in z (top left panel) while the household surplus is decreasing (top right panel). The figure shows the surpluses as functions of the permanent income component for a fixed current wealth level $\bar{b}$ and fixed desired asset position $\bar{b}'$ at various levels of the transitory component. The intuition is the same as for the case of b.

Given the menu $\phi(b')$, a household of type (b, z) must then choose her desired level of renegotiated asset holdings b'. The way in which the household makes her choice is characterized in the following proposition.

Proposition 4.4 *A solution $b'(b, z; \Psi_t), \phi(b, z; \Psi_t)$ to the contracting problem exists. Furthermore,*

(a) If the household's value function $V_t(b, z; \Psi_t)$ is concave in b, such solution is unique.

(b) If the household's value function $V_t(b, z; \Psi_t)$ is differentiable in b, the optimal renegotiated level of asset holdings $b'(b, z)$ is characterized by

$$\frac{\partial_b V_t(b', z; \Psi_t)}{u_c(c_t^R, l_t^*)} = 1 - r_t \tag{4.3.13}$$

Proposition 4.4 simply states the familiar marginal benefit equals marginal cost condition. The household will choose the asset level b' to ensure that the benefit of an additional unit of asset (given by the derivative of the continuation value) equals its cost (which is the marginal utility of current consumption times the number of effective consumption units foregone). Equation (4.3.13) is written so that the marginal benefit (the left hand side) and the marginal cost (the right hand side) are both given in terms of current consumption units. Given the continuous time nature of the problem, for an additional unit of deposits, the household effectively gives up $1 - r_t$ units of current consumption as the interest payment is instantaneous.

Recall that the fee menu offered by the branch implies the household's marginal utility of current consumption is related to the effective household's and branch's surpluses as given by equation (4.3.11). One can use this relationship to rewrite (4.3.13) as:

$$\frac{\partial_b V_t(b', z; \Psi_t)}{u_c(\tilde{c}_t, l_t^*)} = (1 - r_t)\, f(b, z, b'; \Psi_t), \tag{4.3.14}$$

where $\tilde{c}_t = c_t^{\text{NR}} - (1-r_t)(b'-b)$ and $f(b,z,b';\Psi_t) \equiv \left(\frac{\theta}{1-\theta}\right)\widetilde{CS}_t(b,z,b';\Psi_t)\cdot\widetilde{BS}_t(b,z,b';\Psi_t)^{\gamma-1}$. Note that $\tilde{c}_t$ represents the household's consumption level if the branch simply "transfers" its cost of funds to the household; the borrowing/lending rate equals the interest rate the branch pays to the main bank.[104] Therefore, equation (4.3.14) implies that the effect of the financial friction can be thought of as being entirely captured by a wedge in the cost of funds, which is represented by $f(b,z,b';\Psi_t)$. Importantly, this wedge is *heterogenous* as it depends on the households current wealth b, income level z, and desired asset holdings b'. In the absence of such a wedge, (i.e. $f(b,z,b';\Psi_t) = 1$), the cost of funds is equal to $1 - r_t$ for *all* agents in the economy; which is the standard case of a perfectly elastic supply of funds.

What determines the size of the wedge? To answer this question note that the total household's surplus from the contract is given by $CS_t = \widetilde{CS}_t + u(c_t^{\text{R}}, l_t^*) + (1-\gamma)^{-1}$. Therefore, define $\widehat{CS}_t \equiv \widetilde{CS}_t + u(\tilde{c}_t, l_t^*) + (1-\gamma)^{-1}$; the total surplus the household would receive from the banking service if the branch would simply set $\tilde{r}_t = r_t$. In essence, $\widehat{CS}_t$ captures the intrinsic value that the banking service has for the household. It is easy to see that $f(b,z,b';\Psi_t) = 1$ if and only if $\widehat{CS}_t = 0$. That is, if the banking service has no intrinsic value for the household, then the bank can't extract any surplus from her and thus simply charges her the prevailing market interest rate. It follows that $f(b,z,b';\Psi_t) \geq 1$ if and only if $\widehat{CS}_t \geq 0$. Whenever the banking service has intrinsic value for the household, the branch can extract some of this value and thus creates a wedge that magnifies the cost of funds for the households. Note that the argument is symmetric in that $f(b,z,b';\Psi_t) < 1$ if and only if $\widehat{CS}_t < 0$. When the banking service has no intrinsic value at the *prevailing* market rate, the branch is willing to reduce the cost of funds the household faces in order to make it attractive for her to engage in the service.[105]

I next characterize the behavior of the wedge across the different households in the economy. The bottom left panel of Figure 4.1 shows the wedge $f(b,z,b';\Psi_t)$ as a function of the desired level of asset holdings b' for a household with a fixed productivity level $\bar{z}$ at various levels of current wealth b. As it can be seen from the figure, the wedge is concave in b'. From Proposition 4.3, the household's effective surplus $\widetilde{CS}_t$ is increasing in b' while the branch's effective surplus $\widetilde{BS}_t$

[104] Recall the definition of $\phi_t \equiv r_t b - \tilde{r}_t b'$. The branch simply "transferring" its cost of funds to the household implies $\tilde{r}_t = r_t$ and so $\phi_t = r_t(b-b')$ and $BS_t = 0$.

[105] In particular, the branch can charge a lower interest rate than the prevailing market rate as long as $BS_t \geq 0$, which in turn requires $b' > 0$.

is decreasing. Intuitively, the household's value for the service increases as the asset adjustment allows her to have a larger continuation value; hence allowing the branch to increase the wedge. However, by increasing her asset holdings, the household is already transferring resources to the branch via deposits; which decreases the branch's incentive to get additional resources via the fee and hence decreases the wedge. Due to the concavity of the value function, for relatively low values of b' the dominant effect is that of $\widetilde{CS}_t$ while the converse is true for relatively large values of b'.

The wedge $f(b, z, b'; \Psi_t)$ as a function of the household's current asset holdings b, given a fixed productivity level $\bar{z}$ and various levels of the readjusted asset position b', is shown in the bottom left panel of Figure 4.2. The wedge is concave in b. Again, from Proposition 4.3, the household's effective surplus $\widetilde{CS}_t$ is decreasing in b while the branch's effective surplus $\widetilde{BS}_t$ is increasing. As the wealth level of the household increases, her valuation of the service decreases. As she is wealthier, she can have a larger continuation value *and* current consumption without having to readjust her asset position. This implies a smaller wedge. For the branch, meeting a wealthier household means it can extract more surplus via the fee, allowing for a larger wedge. For this case, the effect of $\widetilde{CS}_t$ is dominant for relatively large values of b. A similar argument explains why the wedge is also concave in the productivity level z, as show in the bottom left Figure 4.3.

Finally, given the wedge implied by the financial friction, the household's optimal choice of adjusted asset holdings is characterized in the next proposition.

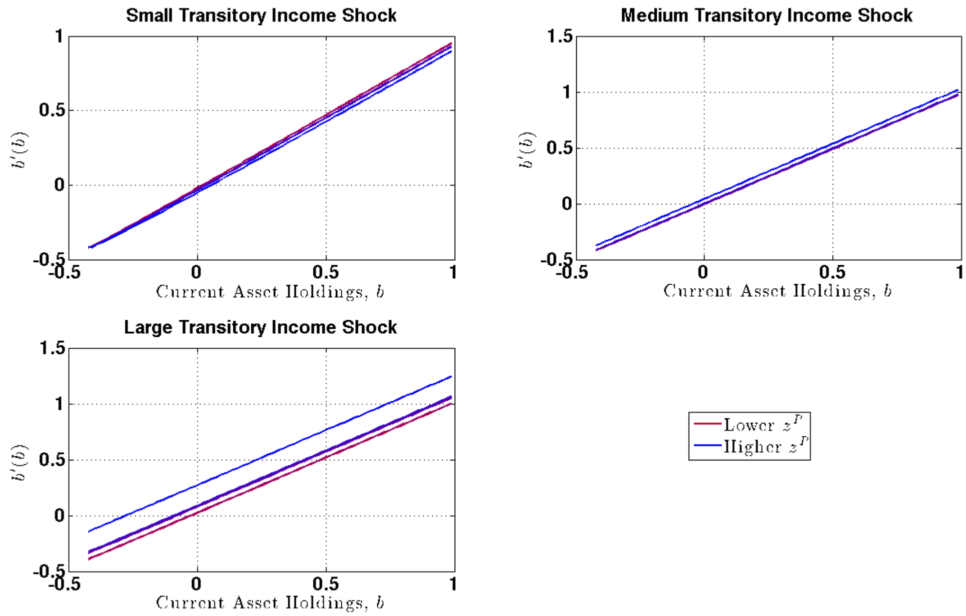
Proposition 4.5 *Consider a household of type (b, z). Her optimal choice of adjusted asset holdings $b'(b, z)$ is a monotone non-decreasing function of b for all z.*

Figures 4.4 - 4.6 illustrate the outcome of the optimal contract given the baseline calibration of the model. Figure 4.4 shows the optimal level of adjusted asset holdings $b'(b, z)$. Each panel corresponds to a different level of the transitory income component. As stated in Proposition 4.5, the optimal level of adjusted asset holdings is increasing in the level of current wealth regardless of the income level. As the household gets wealthier, she can save more and consume more (i.e. the marginal propensity to consume is less than one).[106]

However, for a fixed level of current wealth, whether $b'(b, z)$ is increasing or decreasing in the permanent income component, z^P, depends on the transitory income state, z^T. Given the consump-

[106]Note that although it is hard to see in the figure, for very small asset holdings the households might be at the borrowing constraint.

Figure 4.4: Optimal Renegotiated Level of Asset Holdings $b'(b, z)$

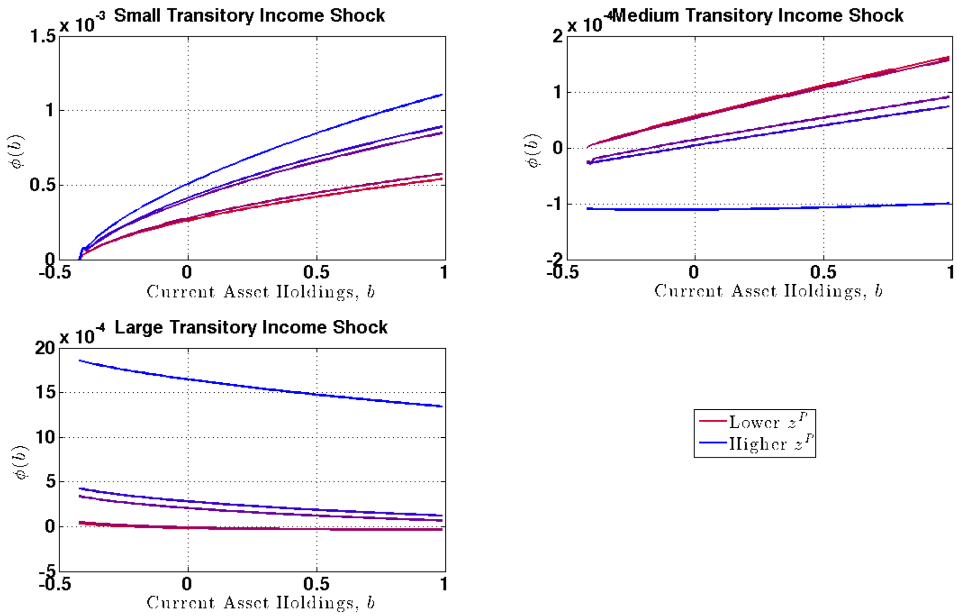


tion smoothing behavior of the household, a small transitory shock would increase the household's incentive to borrow whereas a large transitory shock makes her more prone to save. This behavior is based on the expectation that her income would revert towards the mean in the near future (given the transitory nature of the shock). However, the realization of a bad or good permanent income shock can counter this incentive. On one hand, as shown in the top left panel of Figure 4.4, within the households with a low temporary shock (prone to borrow), the ones who have a low permanent income realization have an incentive to save (borrow less) since their income in the medium/long term might remain low.[107] On the other hand, within the households with a large transitory shock (prone to save), those who have a low permanent income realization now have a smaller incentive to save (the low permanent shock introduces some incentive to borrow), as shown in the bottom left panel of Figure 4.4. In short, having a larger permanent income shock allows the households to better adjust their behavior to smooth the transitory shocks. If they have an incentive to borrow (lower transitory shock), they can borrow more; conversely, if they have an incentive to save (higher transitory shock), they can save more.

The optimal fee $\phi(b, z)$ is shown on Figure 4.5. Consider first the behavior of the fee as a func-

[107] Given the continuous time formulation, the difference between the permanent and transitory components is not only the magnitude of the shocks but also the frequency -and hence duration- of each of them.

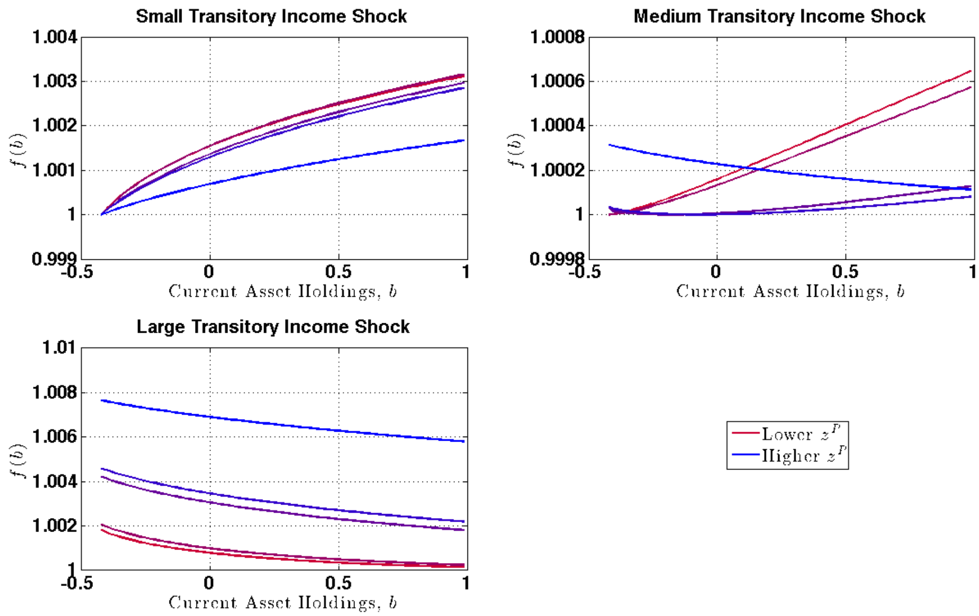
Figure 4.5: Optimal Fee $\phi(b, z)$



tion of a household's wealth level. For the small and medium transitory income levels (top panel), the dominant effect is that of the effective bank surplus; the fee is increasing in the wealth level. As the level of wealth increases, the branch can extract more resources from them and thus charges them a higher fee. For the large transitory income level (bottom left panel), the dominant effect is that of the effective household's surplus; the fee is decreasing in the wealth level. Households with a higher wealth level have a smaller valuation of the banking service and thus the branch must charge them a smaller fee to encourage them to agree to the contract.

Consider next the behavior of the fee as a function of the income state. As the left panels of Figure 4.5 show, the fee is increasing in the permanent income component $\left(z^P\right)$ for the small and large transitory components. Due to the consumption smoothing incentive, households in these two extreme transitory income states find the banking service relatively valuable. In particular, households with a larger permanent component are those who find the service the most valuable. In addition, a larger permanent income implies these households have more resources which the bank can extract using the fee. The combination of these two factors explains why the fee is increasing in z^P for the small and large transitory income states. However, this is not the case for the medium transitory income state; for this state the fee is decreasing in z^P. The intuition is that households don't value an adjustment in their asset position as much in this state; there is no extreme transitory

Figure 4.6: Cost of Funds Wedge Implied by the Optimal Contract $f(b, z; b'(b, z))$



shock to smooth out. Within the households with a medium transitory income shock, those with a higher permanent income find the banking service particularly unattractive and the branch must in turn charge them a smaller fee.

Finally, the cost of funds wedge $f(b, z, b'(b, z); \Psi_t)$ is shown if Figure 4.6. Note first that the wedge is always larger than one regardless of the income and wealth levels; the cost of funds is magnified for *all* households when the financial friction is introduced. Second, for the large transitory income state, the behavior of the fee and the cost of funds wedge is identical; both are decreasing in the wealth level and increasing in the permanent income state. This is a consequence of both objects being determined mostly by the effective household and branch surpluses. Third, for the small transitory income state, the fee and cost of funds wedge are both increasing in wealth. However, while the fee is also increasing in the permanent income level, the cost of funds wedge is decreasing. For the small transitory income state, the contemporaneous marginal utility of households plays a non-trivial role in the determination of the fee (see equation (4.3.11)). For households with a higher level of permanent income, the contemporaneous marginal utility is relatively small (due to their larger consumption). This implies that the bank can extract a large amount of resources by charging a higher fee without distorting their inter-temporal decision too much. Lastly, for the medium transitory income state, the behavior of the fee and the cost of funds wedge is identical for sufficiently low levels of the permanent income state; both objects are increasing in wealth and decreasing in permanent income. However, for large enough permanent income levels the marginal utility of current consumption plays again a non-trivial role in the determination of the contracting fee. As the contemporaneous marginal utility is relatively small (given larger permanent income implies larger consumption), the branch can extract a large amount of resources by charging higher fees without distorting the household's inter-temporal decision as much; the fee is slightly increasing in the level of wealth while the cost of funds wedge is non-increasing.

4.4 Heterogeneous Bank Pass-Through Mechanism

My model features an heterogeneous bank pass-through of credit expansions; banks not only charge differentiated credit conditions across households in the economy but the *change* in such credit conditions following a credit expansion is non-homogenous across them. This novel feature of my model is what allows me to match the empirical distribution of households' borrowing adjustments following a credit expansion as documented by Agarwal et al. (2016).

In the context of my model, a credit expansion is given by a decrease in a branch's cost of funds. Note that a branch's cost of funds is just the real interest rate the branch must pay the main bank (i.e. the opportunity cost of lending the funds to private households). With this in mind, the following proposition characterizes the households' borrowing adjustments following a credit expansion.

Proposition 4.6 *Consider a household with asset holdings b and productivity z. The household's effective surplus $\widetilde{CS}_t(b, z, b'; \Psi_t)$ is given by* (4.3.9) *and the branch's effective surplus $\widetilde{BS}_t(b, z, b'; \Psi_t)$ is given by* (4.3.12)*. Then*

$$\frac{\partial b'_t(b, z; \Psi_t)}{\partial r_t} = \left(1 + \boldsymbol{\Gamma(b, z, b'; \Psi_t)}\right) \cdot \frac{\boldsymbol{\partial \phi_t(b, z, b'; \Psi_t)}}{\boldsymbol{\partial r_t}} \cdot \left(\frac{\boldsymbol{\partial \phi(b, z, b'; \Psi_t)}}{\boldsymbol{\partial b'}}\right)^{-1} ; \quad (4.4.1)$$

where:

1. $\boldsymbol{\Gamma(b, z, b'; \Psi_t)} \equiv \dfrac{(1 - r_t)(b' - b)}{\widetilde{BS}_t(b, z, b'; \Psi_t)}$

2. $$\frac{\partial \phi_t(b,z,b';\Psi_t)}{\partial r_t} \equiv \frac{1}{\gamma\,(1-r_t)}\left(\frac{\widetilde{BS}_t(b,z,b';\Psi_t)}{\widetilde{CS}_t(b,z,b';\Psi_t)}\right)^{1/\gamma}$$

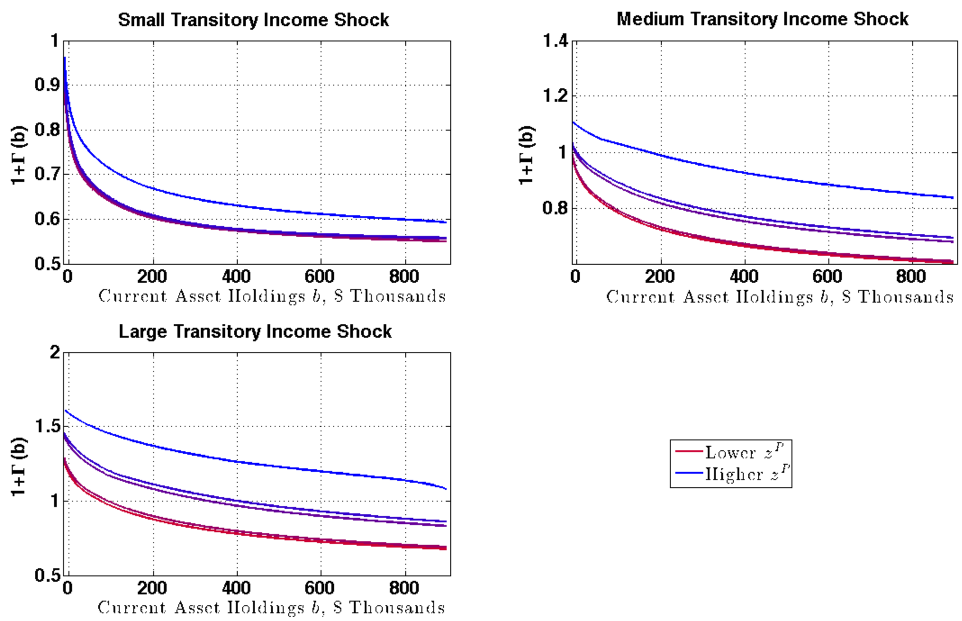
3. $$\frac{\partial \phi_t(b,z,b';\Psi_t)}{\partial b'} \simeq 1 \;\forall\, b,z$$

There are two subtle points that need to be clarified about Proposition 4.6. First, it characterizes the response of households' borrowing to a change in a branch's cost of funds. Therefore, it *does not* incorporate the wealth effect resulting from changes in the real interest rate that affect the households' *existing* level of wealth. Given the continuous time nature of the model, households earn (or pay) the prevailing market interest rate r_t on their existing wealth at each instant in time. However, whenever a renegotiation opportunity arises, they earn (or pay) a one-time interest rate negotiated between the branch and the household, $\tilde{r}_t\,(b,z;\Psi_t)$. Thus, strictly speaking, a change in the real interest rate r_t affects households' borrowing via the wealth effect and the cost of funds effect. Proposition 4.6 just refers to the latter. Second, equation (4.4.1) is evaluated at the optimal level of borrowing $b' = b'(b,z;\Psi_t)$.

As it is evident from Proposition 4.6, there are three factors that affect the response of households' borrowing. First, the amount of surplus that a bank is able to extract from households; the term $1+\Gamma(b,z,b';\Psi_t)$ which I refer to as the "pass-through". Second, the way in which the branch adjusts the credit conditions in response to a change in its cost of funds; the branch's marginal propensity to lend given by the term $\frac{\partial \phi_t(b,z,b';\Psi_t)}{\partial r_t}$. Lastly, the way in which households' adjust their borrowing in response to the adjustment in the credit conditions; the households' marginal propensity to borrow given by the term $\left(\frac{\partial \phi_t(b,z,b';\Psi_t)}{\partial b'}\right)^{-1}$.

Consider first the term $1+\Gamma(b,z,b';\Psi_t)$. Note that $\Gamma(b,z,b';\Psi_t)$ is negative for all households who are effectively withdrawing funds from the branch (i.e. $b' < b$). Households must rely on bank branches to adjust their asset position, which effectively gives the latter some monopolistic power. Thus, for any level of desired asset holdings b', the branches use this monopolistic power to extract some of the surplus households would get from a decrease in the credit costs; for every \$1 decrease in its cost of funds, the branch decreases the credit costs for *all* households who are withdrawing funds (effectively borrowing) by less than \$1. Thus this term captures the "pass-through" of the

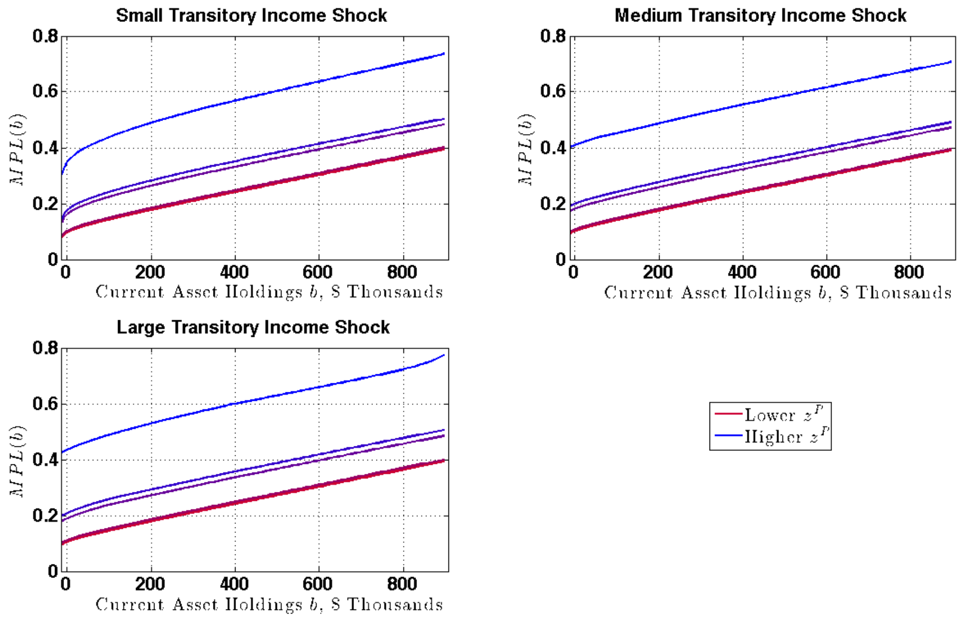
Figure 4.7: Pass-Through $1 + \Gamma(b, z)$



credit expansion. It is worth pointing out that there is some degree of heterogeneity in this "pass-through". On one hand, households from whom the bank is already extracting a large amount of resources, large $\widetilde{BS}_t(b, z, b'; \Psi_t)$, get a larger "pass-through". On the other hand, household's with a larger borrowing adjustment $(b' - b)$ get a smaller "pass-through". Figure 4.7 shows the "pass-through" as a function of the wealth level b for different levels of the transitory (z^T) and permanent (z^P) income states. Given the model's calibration, these two effects operate in opposite directions and the result is that the "pass-through" is decreasing in wealth and income; despite having a larger bank's effective surplus $\widetilde{BS}_t(b, z, b'; \Psi_t)$, higher wealth/income households make the largest borrowing adjustments resulting in a smaller "pass-through".

Consider next the marginal propensity to lend (MPL). Most of the heterogeneity in households' borrowing adjustments stems from the MPL. Figure 4.8 shows the MPL as a function of the wealth level b for different levels of the transitory (z^T) and permanent (z^P) income states. As shown on the figure, branches are willing to relax the credit conditions more (i.e. decrease the fee by more) for households with higher wealth and income. From Proposition 4.6, the marginal propensity to lend is positively correlated with the effective branch's surplus $\widetilde{BS}_t(b, z, b')$ and negatively correlated with the effective household's surplus $\widetilde{CS}_t(b, z, b')$. Thus, given the discussion of Section 4.3.5, it

Figure 4.8: Marginal Propensity to Lend MPL(b, z)

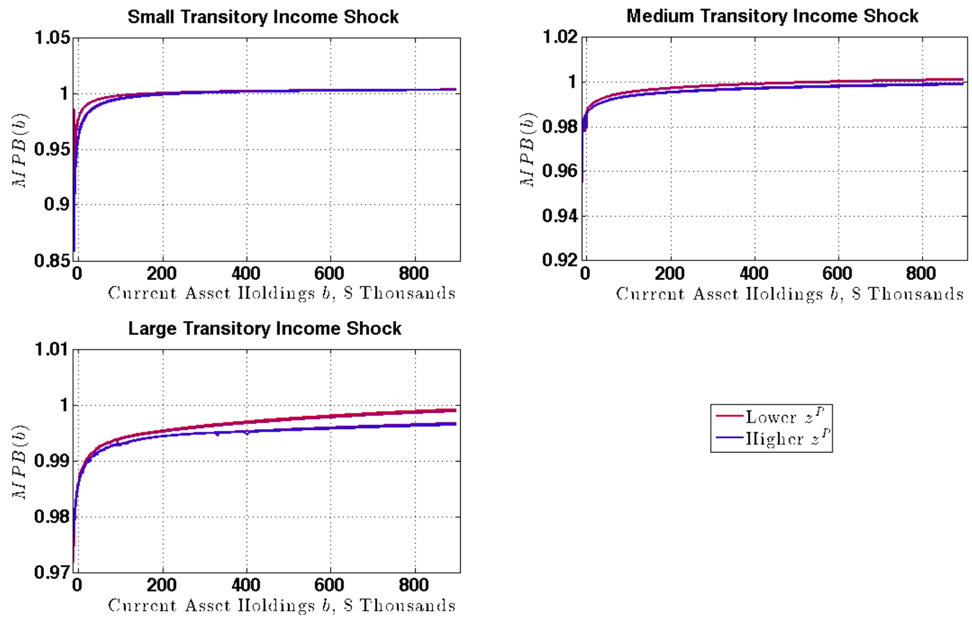


is easy to see that the MPL is an increasing function of the wealth and income levels.[108] Intuitively, the branch must decrease the fee more for those households who have the smallest valuation of the service, small $\widetilde{CS}_t(b, z, b')$, in order to make them willing to use the service. Additionally, the branch is willing to decrease the fee more for those households with a large $\widetilde{BS}_t(b, z, b')$ as it can extract more resources from them. Thus, in light of Propositions 4.3 and 4.5, the marginal propensity to lend is increasing in wealth and income; households with large wealth and income are precisely those who have the smallest $\widetilde{CS}_t(b, z, b')$ and largest $\widetilde{BS}_t(b, z, b')$. It is important to note that, despite the heterogeneity, the marginal propensity to lend is positive for *all* wealth and income levels; a decrease in the cost of funds leads to a relaxation of the credit conditions for all households in the economy.

The third factor affecting the adjustment in borrowing following a decrease in the branch's cost of funds is the households' marginal propensity to borrow (MPB). Figure 4.9 presents the MPB as a function of the wealth level b for different levels of the transitory (z^T) and permanent (z^P) income states. As shown on the figure, the marginal propensity to borrow is pretty close to one for all households in the economy. That is, households increase their borrowing almost one-for-one with a

[108]Note that although I abstract from modeling default in my model, including it would just enhance the mechanism at work. The cost of default would decrease the marginal propensity to lend as it would have a negative impact on the amount of resources the bank can extract from households.

Figure 4.9: Marginal Propensity to Borrow MPB(b, z)



decrease in the contracting fee. For those households at (or near) the borrowing limit $\underline{b}$, the marginal propensity to borrow is somewhat smaller than one. Intuitively, these households can't increase their borrowing one-for-one given the binding borrowing limit. Thus, although the marginal propensity to borrow affects households' borrowing response to a change in the cost of funds, its contribution to the *heterogeneity* of such response is negligible.

In the end, in my model the households' borrowing response following a credit expansion is mostly driven by two factors; the "pass-through" and the "heterogeneous adjustment" of the contracting fee across households. The former refers to the observation that the branches decrease the credit costs for households by *less* than the decrease in their cost of funds. The latter refers to the fact that the *change* in the credit costs is heterogeneous across the population as it depends on factors such as households' valuation of the banking service and the amount of resources households can offer to banks.

4.5 Calibration

The following section details the strategy used in choosing the parameter values for the model. There are three main objectives of the calibration strategy. First, to calibrate the exogenous id-

Table 4.2: Earnings Process Parameter Estimates

Parameter	Transitory Component j=1	Permanent Component j=2
Arrival rate (λ_j)	0.08	0.007
Mean reversion (β_j)	0.76	0.009
Std. Deviation (σ_j)	1.74	1.530

Note: The parameter estimates correspond to those presented in Kaplan, Moll and Violante (2016). All rates are expressed as quarterly values.

iosyncratic productivity process so that it reflects the leptokurtic nature of the U.S. income changes recently documented by some empirical literature. Second, to develop a mapping between the variables in the model and the U.S. FICO score distribution. Third, to be able to match some key moments of the U.S. asset distribution.

4.5.1 Idiosyncratic Productivity Process

I use the idiosyncratic productivity process estimated by Kaplan, Moll and Violante (2016); which assumes log-earnings is given as the sum of two independent components; $\log z_t = z_t^1 + z_t^2$. In turn, each component is given by a continuous-time continuous-state process of the form

$$dz_t^j = -\beta_j z_t^j dt + \epsilon_t^j dN_t^j; \tag{4.5.1}$$

where $\epsilon_t^j \sim N\left(0, \sigma_j^2\right)$ and dN_t^j is a pure Poisson process with arrival rate λ_j. The estimated parameter values are summarized in Table 4.2.

I choose this type of idiosyncratic income process for two reasons. First, it resembles the standard discrete time specification where log-earnings is modeled as the sum of a transitory and a permanent components; the process in equation (4.5.1) closely resembles a discrete time AR(1) process. Second, this formulation allows for the arrival of each income innovation to be stochastic instead of deterministic. The inclusion of the Poisson process implies that the income innovations are stochastic with an arrival rate of λ_j instead of being realized at every instant in time.[109]

When solving the household's problem, I approximate the continuous-time continuous-state processes in equation (4.5.1) with a continuous-time discrete-state processes. For each of the two

[109] As Kaplan, Moll and Violante (2016) argue, the frequency of earning shocks plays a crucial role in any model of households' portfolio choice.

components $(j = 1, 2)$ I proceed in two steps. First, I construct a grid z_j with 5 grid points for the persistent component and 3 grid points for the transitory component. The lower bound, upper bound, and spacing between grid points, are all parameters to be determined by the calibration. Second, I construct the associated continuous time transition matrix based on a finite difference approximation of the continuous-state process.

I calibrate the upper bound, lower bound, and spacing between grid points so that the annual moments produced by simulating the discrete-state process match eight key annual earnings moments from the continuous-state process as reported in Kaplan, Moll and Violante (2016). These moments are presented in Table (4.3). As it can be seen from the table, the discrete-state specification does a fairly good job in approximating the continuous-time one in terms of matching these moments.

4.5.2 FICO Score Distribution

The results in Agarwal et al. (2016) about the distribution of changes in borrowing following credit expansions are presented in terms of households' FICO scores. Since I will be comparing the predictions of my model with these results, I need to map the wealth and income distribution implied by my model into the FICO score distribution of the U.S. The way in which I construct this mapping is guided by the way in which I view FICO scores. First, I interpret them as just providing an ordering among households in the population (in terms of default risk). Second, I conjecture that the main factors that are considered when constructing FICO scores are: (i) total amount owed by a household, (ii) household's payment history, (iii) length of a household's credit history, (iv) composition of the household's debt, and (v) new credit obtained by the household. With this in mind, I use some of my model's variables to construct a statistic that satisfies this two criteria; it ranks households according to their likeliness to default and it incorporates some of these factors that are considered when constructing the actual FICO scores (in particular (i) and (v)). I then construct a mapping from this statistic to the U.S. FICO score distribution.

Recall that for a household of type (b, z), the optimal outcome of the contract is given by the desired level of asset holdings $b'(b, z)$ and the fee $\phi(b, z; b'(b, z)$.[110] Suppose there was the option of defaulting. Assume that if a household chooses to default, she does not repay her current debt

[110] Throughout this section, I drop the dependence of the variables on time since the calibration targets the steady state outcomes. Instead I use the notation x_{ss} to refer to the steady state value of variable x.

Table 4.3: Earnings Process Target Moments

Moment	Data	Continuous-State	Discrete-State
Variance of log earnings	0.70	0.70	0.72
Variance of 1yr change	0.23	0.23	0.27
Variance of 5yr change	0.46	0.46	0.44
Fraction 1yr $< 10\%$	0.54	0.56	0.57
Fraction 1yr $< 20\%$	0.71	0.67	0.68
Fraction 1yr $< 50\%$	0.86	0.85	0.83

Note: The moments corresponding to "Data" and the "Continuous-State" model correspond to those presented in Kaplan, Moll and Violante (2016).

nor the interest. In turn, the branch refuses to negotiate a new contract and the household is unable to get any new funds. Therefore, the benefit the household gets from defaulting is given by

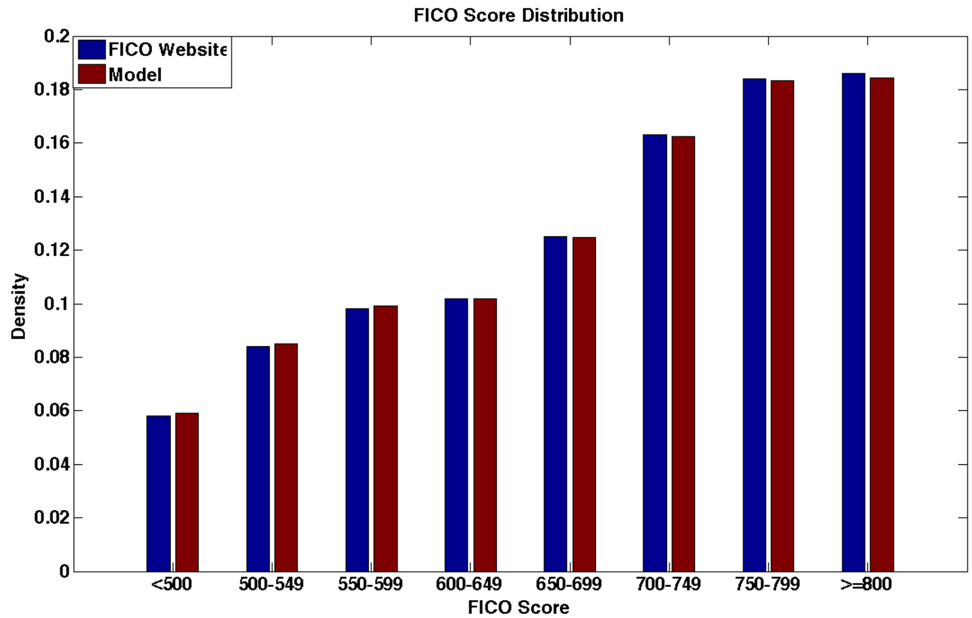
$$S_{\mathrm{D}}(b,z;b'(b,z)) = \left[u\left((1-\tau^l_{ss})w_{ss}zl^*_{ss}, l^*_{ss}\right) + V(0,z)\right] - \left[u\left(c^{\mathrm{R}}(b,z), l^*_{ss}\right) + V(b'(b,z),z)\right]. \tag{4.5.2}$$

The first term is simply the total utility (flow plus continuation value) if the household chooses to default. The second term capture the total utility the household gets if she repays and hence gets the optimal contract. The household would find default attractive as long as $S_{\mathrm{D}}(b,z;b'(b,z)) \geq 0$. I use this idea to construct a summary statistic that "ranks" households in the economy according to their incentive to default:

$$\widehat{FICO}(b,z,b'(b,z)) = -\frac{S_{\mathrm{D}}(b,z;b'(b,z))}{\max_{b,z} S_{\mathrm{D}}(b,z;b'(b,z))} \tag{4.5.3}$$

The summary statistic defined by (4.5.3) measures the incentive to default of a household of type (b,z) *relative* to the household who has the largest incentive to default. Note that given this definition, $\widehat{FICO}(b,z,b'(b,z)) \in [-1, \underline{\mathrm{M}}]$, where $\underline{\mathrm{M}} \geq -1$. Therefore, for two households (b_1,z_1) and (b_2,z_2), household 1 finds default more attractive (is more likely to default) than household 2 if and only if $\widehat{FICO}(b_1,z_1,b'(b_1,z_1)) < \widehat{FICO}(b_2,z_2,b'(b_2,z_2))$. The sign of the summary statistic is not relevant for my purpose. A non-negative value of the statistic implies that the household does not find default attractive. However, I am only interested in "ranking" the households. That is, given two households who find default unattractive, which one finds it *more* unattractive. If one had to pick the most trustworthy household out of two, one with a FICO score of 761 and another one with

Figure 4.10: FICO Score Distribution

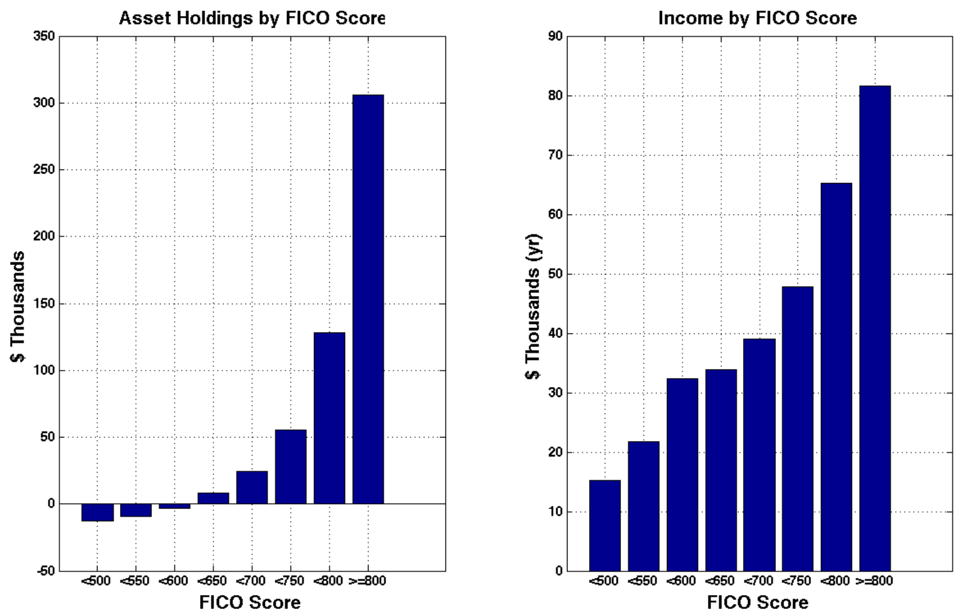


a score of 762, one would probably select the one with the highest FICO score although both have "excellent" credit (score above 750).

In light of the previous discussion, the summary statistic in (4.5.3) captures a household's default risk. This default risk is associated only with two of the five factors that might enter the actual FICO score calculation: the total amount owed by the household (her current holdings b) and her new credit (her desired level of asset holdings b'). However, and to the extent that a household's income is a proxy for her payment history or length of credit history, I am indirectly capturing such factors as well.

To map the model's statistic given in (4.5.3) into actual FICO scores, I define seven threshold values $x_i \in [-1, \underline{\mathrm{M}}]$, where $x_i \leq x_{i+1}$. Each of these values corresponds to a FICO score cutoff: $c_1 = 500$, $c_2 = 550$, $c_3 = 600$, $c_4 = 650$, $c_5 = 700$, $c_6 = 750$, and $c_7 = 800$. Therefore, the threshold values imply FICO score bins. I select the threshold values so that the mass of people in each FICO score bin implied by the invariant distribution of the model matches the percentage of the U.S. population with that range of FICO scores. That is, the threshold values are selected

Figure 4.11: Average Wealth and Income by FICO Score Group



targeting the following moments:

$$M_i = P(c_{i-1} < \text{FICO} \leq c_i) - \int_{(b,z)} \mathbb{1}_{\left\{x_{i-1} < \widehat{FICO}(b,z,b'(b,z) \leq x_i\right\}} dG(b,z), \quad i = 1, 2, ..., 8 \quad (4.5.4)$$

where $x_0 = -1$ and $x_8 = \underline{\mathrm{M}}$, with corresponding FICO score cutoffs $c_0 = 300$ and $c_8 = 850$. Note that $P(c_{i-1} < \text{FICO} \leq c_i)$ denotes the fraction of the U.S. population with a FICO score between c_{i-1} and c_i.[111]

Figure 4.10 shows the distribution of FICO scores across the U.S. population (blue bars) and the one implied by my model (red bards). Additionally, Figure 4.11 shows the average wealth level (left panel) and the average (yearly) income level within each FICO group as implied by the model. Note that the model predicts a positive correlation between FICO scores and income/wealth: households with higher FICO scores are those who, on average, have a higher wealth and income.

[111] I use the data from the FICO$_{\text{TM}}$ website: http://www.fico.com/en/blogs/tag/score-distributions/.

4.5.3 Asset Distribution And Other Parameters

The calibration strategy follows closely Kaplan, Moll and Violante (2016). This is to ensure that, when comparing the model with the heterogenous bank pass-trough to the model without it (standard HANK), any difference in the response of aggregate consumption to a credit expansion is attributable to the pass-through mechanism and not to the model's calibration.

The model parameters that determine the asset distribution are the discount rate ρ, the risk aversion γ, the renegotiation frequency σ, and the borrowing limit $\underline{b}$. The values of these parameters for the calibrated version of the model are shown in Table 4.5. In addition, Table 4.5 shows the Frisch elasticity ν, the disutility of labor scaling parameter ψ, the bargaining parameter θ, the labor income tax rate τ_l, and the lump sum transfer T_{lump}.

The parameters affecting the wealth distribution are calibrated so that the model's invariant wealth distribution matches three target moments of the U.S. wealth distribution: (i) the average private (revolving) consumer debt, (ii) the average asset holdings (liquid and illiquid), and (iii) the fraction of population with debt (non-positive asset holdings). Table 4.4 shows these moments and their model counterparts. Note that this calibration implies that 9% of the population is at the borrowing limit.

The other calibrated parameters are the disutility of labor ψ and the lump sum transfer T_{lump}. The disutility of labor is set so that effective hours worked by households in steady state is equal to a third. The lump sum transfer is calibrated to imply that 40% of households receive a net transfer from the government in steady state.

Finally, the labor income tax rate is set at 25%. It must be noted that the branch's bargaining power θ is completely determined by the risk aversion parameter given the assumption in Proposition 4.2.

4.6 Results

The following section discusses the main results of the paper. First, I present the empirical evidence on the distribution of borrowing adjustments across households following credit expansions as documented by Agarwal et al. (2016) and show that my model can match it. Next, I use my model to show how this mechanism dampens the responsiveness of aggregate consumption to monetary

Table 4.4: Target vs. Calibrated Moments of the Wealth Distribution

Target Moment	Data	Model
Avg. private debt	-0.03	-0.07
Avg. asset holdings	3.18	3.11
Fraction of the population with debt	0.25	0.23
Fraction of the population with net transfer	0.40	0.41

Note: The average asset/debt values are expressed as ratios to annual output. Data is based on SCF 2004 and the Congressional Budget Office (2013).

Table 4.5: List of Calibrated Parameter Values

	Description	Value	Source
r	Return on asset (pa)	2%	Kaplan et al. (2016)
ρ	Discount rate (pa)	4.68%	Calibrated
γ	Risk aversion	1.05	Calibrated
ν	Frisch elasticity	0.5	Kaplan et al. (2016)
ψ	Disutility of labor	19	Avg. hrs worked equal to 1/3
$\underline{b}$	Borrowing limit	-0.42	Equals 1 x labor income
θ	Bank's bargaining power	0.05	By assumption
σ	Renegotiation rate	9	Calibrated (3 times a month)
τ^l	Labor income tax	0.25	Kaplan et al. (2016)
T_{lump}	Lump sum transfer	0.05	Calibrated

Note: Except for r and ρ, all other parameters are given in quarterly terms.

policy expansions.

4.6.1 Distribution of Borrowing Adjustments After A Credit Expansion

Agarwal et al. (2016) use panel data on credit cards in the United States over the period January 2008 to December 2014 to determine the distribution of changes in borrowing across households resulting from credit expansions. Their study groups households in different FICO score bins and quantifies the average change in borrowing for each bin as the product of the Marginal Propensity to Borrow (MPB) and the Marginal Propensity to Lend (MPL). On one hand, the MPL captures the way in which banks adjust the credit card limits for different households in the economy following a credit expansion. On the other hand, the MPB captures the way in which households react and adjust their borrowing following a change in their credit card limits. Therefore, the total effect of a credit expansion in borrowing is just given by the product of these two factors.

Figure 4.12: Results from Agarwal et al. (2016) About The Distribution of Debt Adjustments

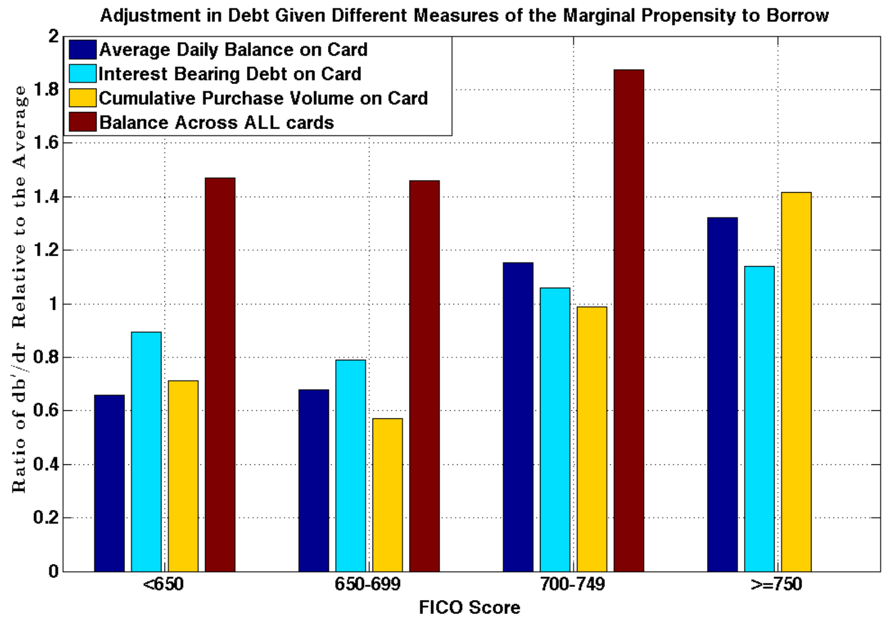


Table 4.1 and Figure 4.12 present a summary of their results. Note that the distribution of borrowing adjustments following a credit expansion depends on the measure used in constructing the marginal propensity to borrow. In particular, Agarwal et al. (2016) use four different measures for "credit card debt" when constructing the marginal propensity to borrow; Average Daily Balance, Interest Bearing Debt, Cumulative Purchase Volume, and Balance Across All Credit Cards. The first three measures ignore portfolio effects; they reflect the total debt balance in the *specific* card for which the credit limit changed. The last measure incorporates portfolio effects; it measures debt as the net borrowing from *all* the credit cards owned by a household. With this in mind, there are three important remarks about these results. First, from Figure 4.12, it is clear that when the borrowing measure does not capture portfolio effects (i.e. debt is measured as Average Daily Balance, Interest Bearing Debt, or Cumulative Purchase Volume) the adjustment in borrowing is increasing in the FICO score. Additionally, for all of these three measures of debt the distribution of borrowing adjustments across FICO scores is fairly similar. Second, when the borrowing measure captures portfolio effects (i.e. debt is measured as the Balance Across *all* Cards) the adjustment in debt is increasing in the FICO score only up to the FICO score bin of $700 - 749$. As Table 4.1 and Figure 4.12 show, for the highest FICO score bin the adjustment in borrowing is zero. Households with a FICO score of 750 and above simply readjust their portfolio; they increase the use of the

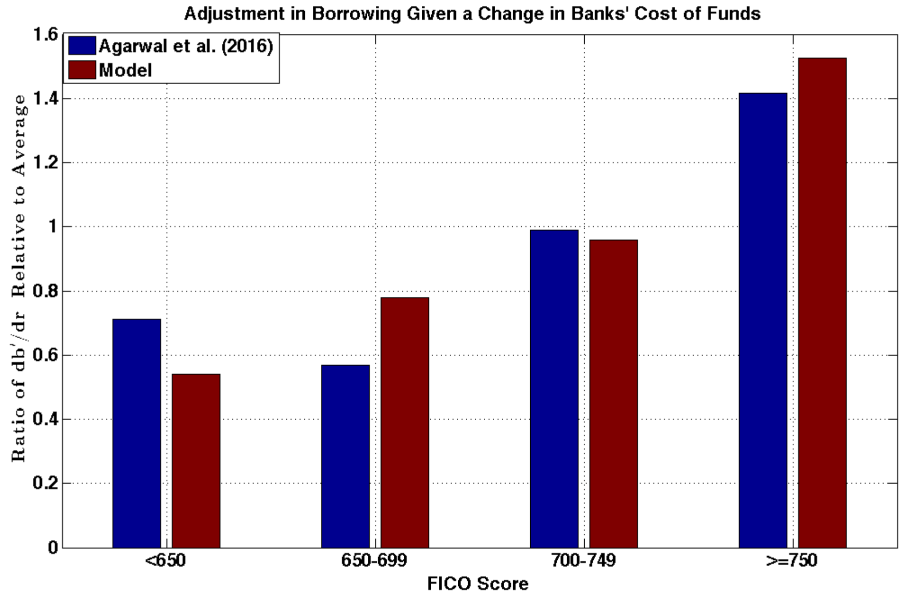
card with the higher credit limit but *decrease* the use of the other credit cards.[112] The result is that there is no increase in net borrowing for these households. Finally, although the study of Agarwal et al. (2016) focuses on credit cards, the authors argue that their results are likely to be present in a wide array of lending/borrowing markets. In the authors' own words: "While the credit card market is of stand-alone interest because credit cards are the marginal source of credit for many U.S. households, mortgage lending and small business lending are other important channels for monetary policy transmission. We think that our finding that the pass-through of changes to banks' cost of funds is muted for less creditworthy consumers ... is likely to apply across this broader set of markets".

Figure 4.13 shows that my model is able to replicate the results presented in Agarwal et al. (2016) about the distribution of changes in borrowing following a credit expansion. The figure shows the change in borrowing by FICO score group relative to the population average. The blue histogram refers to the results from Agarwal et al. (2016) and the red histogram corresponds to my model's prediction. In light of the remarks in the previous paragraph, there are two important observations that the reader must keep in mind. First, I interpret the results in Agarwal et al. (2016) as applicable to the general credit market; my model is by no means a rigorous model about the credit card market but rather a general representation of households' access to credit. Second, my model is a one asset model and thus ignores any portfolio effects; I am able to match the results from Agarwal et al. (2016) when they use *any* of the borrowing measures that does not incorporate such portfolio effects.

The empirical results that my model is able to replicate imply that households with higher FICO scores adjust their borrowing by more following credit expansions; roughly speaking, households at the top of the FICO score distribution increase their borrowing by about $2 - 2.5$ times more relative to those at the bottom. The driving force behind these results can be understood from Table 4.1. In a nutshell, the banks relax the credit conditions more for households with a higher FICO score but these households are precisely the ones who don't use much of the extra credit to increase their borrowing. On one hand, the marginal propensity to borrow is decreasing in the FICO score; for a given increase in credit limits households on the highest FICO score bin increase their borrowing

[112] Other studies such as Díaz-Giménez, Glover and Ríos-Rull (2011) have also found evidence about this portfolio effect.

Figure 4.13: Model's Implied Distribution of Debt Adjustments by FICO Score



Note: The histogram corresponding to Agarwal et al. (2016) is constructed using the Cumulative Purchase Volume as the measure of "credit card debt".

only by about two fifths relative to households on the lowest FICO score bin. On the other hand, the marginal propensity to lend is increasing in the FICO score; banks increase the credit limits by about five times more for the households in the highest FICO score bin relative to those in the lowest FICO score bin. In the end, ignoring portfolio effects, the force that dominates is that of the marginal propensity to lend and households with a higher FICO score end up increasing their borrowing by more.

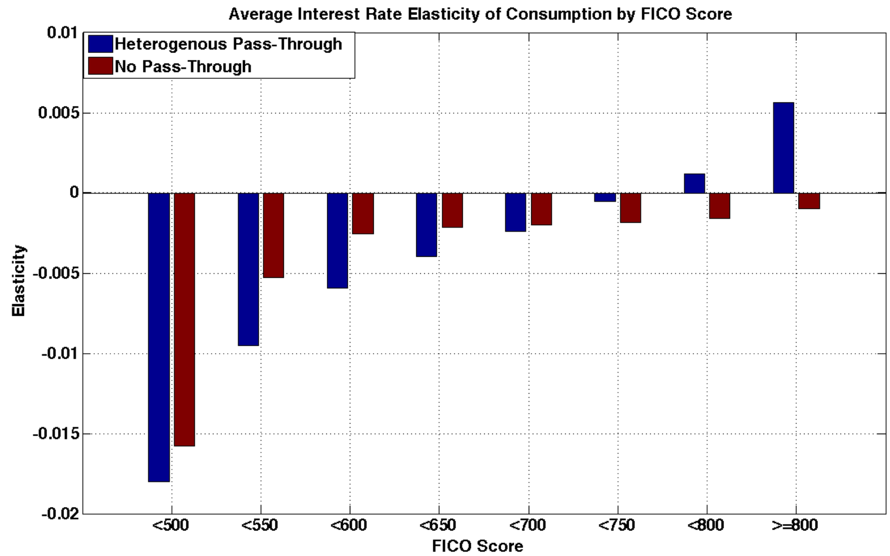
In my model, the mechanism that explains these results is similar; households' borrowing response following credit expansions is driven by the "heterogeneous bank pass-through" mechanism arising from the contracting environment. In Section 4.4, I decomposed the households' borrowing response to a credit expansion into three terms; the "pass-through" term, a term related to the bank's branch marginal propensity to lend, and a term related to the household's marginal propensity to borrow. Given the model's calibration, I showed that the "pass-through" term is decreasing in wealth/income, the marginal propensity to lend is increasing in wealth/income, and the marginal propensity to borrow is pretty much identical and equal to one across households. In the end, the effect that dominates is that of the marginal propensity to lend; the way in which the bank adjusts the credit conditions (in my model the contracting fee). Intuitively, introducing the contracting environment gives the bank some market power over the household; the bank then uses the contracting fee as a tool to extract surplus from households. Two factors affect the way in which the bank makes adjustments to the contracting fee. First, the amount of surplus the bank can actually extract from the households (i.e. the bank's value of the banking service). Second, the amount of surplus that households get from the transaction (i.e. the households' value of the banking service). In my model, the households with high wealth and income are those from whom the bank can extract the most resources. Additionally, it is precisely these households who value the service the least. As a consequence, the bank *needs* to relax the credit conditions more for the high wealth and income households (in order to make the service attractive for them) and the bank is *willing* to do so (since these households give the bank the largest surplus). The combination of these two effects implies that the marginal propensity to lend is increasing in wealth and income. Finally, in Section 4.5.2 I showed that the mapping from my model to the U.S. FICO score distribution implies that my model's FICO score is increasing in wealth and income. Thus, households with a higher FICO score are precisely those whose borrowing increases the most, just like the evidence in Agarwal et al. (2016) suggests.

4.6.2 Effect on Aggregate Consumption Response

In the previous section, I showed that the "heterogeneous bank pass-through" mechanism that arises in my model allows me to replicate the empirical evidence in Agarwal et al. (2016). I now proceed to analyze the effect that this "heterogeneous bank pass-through" has on the monetary policy transmission mechanism to aggregate consumption. In other words, I use my model as a laboratory to study how the monetary policy transmission mechanism is affected when financial institutions are able to adjust the credit conditions non-homogeneously across different households in the economy.

Figure 4.14 compares the effect of a monetary policy shock on households' interest rate elasticity of consumption for two versions of the economy; one that includes the heterogeneous pass-through mechanism and one that doesn't. The blue histogram illustrates the consumption response across households for the economy that incorporates the heterogeneous pass-through; which is the economy described in Section 4.3. For the version of the economy that does not include the heterogeneous pass-through, given by the red histogram, households can adjust their asset holdings

Figure 4.14: Consumption Elasticity Upon Impact by FICO Score



at any instant in time and get/pay an interest rate equal to the prevailing market rate r_t (i.e. the banking sector can't extract any surplus). In other words, this frictionless version of the economy corresponds to the one (liquid) asset version of the HANK model presented in Kaplan, Moll and Violante (2016). Both versions of the economy are calibrated to target the moments described in Section 4.5.

The interest rate elasticity of consumption measures the percentage change in consumption relative to the percent change in the real interest rate at the instant the initial monetary policy shock hits the economy. I introduce the monetary policy shock following Kaplan, Moll and Violante (2016). In particular, I assume monetary policy is given by a Taylor rule of the form $r_t = r_{ss} + (\phi - 1) \cdot \pi_t + \epsilon_t$; where r_{ss} denotes the steady state real interest rate, π_t denotes the inflation rate at instant t, $\phi > 1$ determines the strength of the monetary authority's response to inflation, and ϵ_t is an unexpected (but deterministic) temporary monetary policy shock. As in the HANK framework, when a surprise monetary policy shock hits the economy there is a general equilibrium response in the path of prices $\Psi^e = \left\{ r_t^e, w_t^e, \tau_t^e(\cdot), \pi_t^e \right\}_{t \geq 0}$. However, as Kaplan, Moll and Violante (2016) note, the direct effect of the change in the real interest rate on consumption can be isolated from the general equilibrium effects by considering the alternative price path $\widehat{\Psi} \equiv \left\{ r_t^e, w_{ss}, \tau_{ss}(\cdot), \pi_t^e \right\}_{t \geq 0}$; where w_{ss} and $\tau_{ss}(\cdot)$ are the steady state wage rate and tax functions. That is, by solving the recursive formulation of the households' problem under the price path $\widehat{\Psi}$, one can focus on the adjustment in households' behavior purely due to changes in the real interest rate and exclude the general equilibrium effects implied by changes in the wage rate and tax functions. Furthermore, given the nature of my exercise, I do not need to close the model to solve for the path of inflation rates $\left\{\pi_t^e\right\}_{t\geq 0}$.[113] Again, appealing to the results presented in Kaplan, Moll and Violante (2016), the equilibrium path of the real interest rate (in their model the return on the liquid asset) is qualitatively identical and quantitatively close to the exogenous path of the Taylor rule innovation ϵ_t. Therefore, I just assume that the "equilibrium" time path for the prevailing market rate is given by $r_t^e = r_{ss} + e^{-\eta t}\epsilon_0$, where ϵ_0 is the initial surprise monetary policy shock.

As seen from Figure 4.14, the consumption elasticity is negative across *all* FICO score groups when the heterogeneous pass-through is not included; a decrease in the real rate leads to an increase in consumption across all FICO groups. However, when the heterogenous pass-through is included, the consumption elasticity is negative only for households with a FICO score below 750. For households with a FICO score above 750, a decrease in the real rate leads to a *decrease* in consumption. In a nutshell, the heterogeneous pass-through enhances the consumption response for households with low FICO scores (i.e. low wealth and income) while muting (even reversing) the consumption response for households with high FICO scores (i.e. high wealth and income).

This result hinges on the effect that the heterogeneous bank pass-through has on the substitution effect of a decrease in the real rate. As in most standard models, the change in the real rate has two effects; an inter-temporal substitution effect and a wealth effect. On one hand, the wealth effect depends on a household's asset position. For households with negative wealth, a decrease in the real rate reduces past debt obligations; their wealth effectively increases providing an additional incentive to consume. For households with positive wealth, a decrease in the real rate reduces the interest accrued on past savings; their wealth effectively decreases leading to a smaller incentive to consume. On the other hand, the inter-temporal substitution effect refers to a household's incentive to increase consumption due to a smaller real rate; which is qualitatively the same for all households (regardless of their wealth and income levels). When the heterogeneous pass-through is included, the substitution effect is enhanced for households with a negative asset position and dampened for households with a positive asset position. Therefore, for households with negative wealth the

[113] Note that the households' and banking sector problems are both entirely cast in real terms.

consumption response is enhanced; the larger substitution effect and the wealth effect both tend to increase consumption given the decrease in the interest rate. For households with positive wealth the consumption response is dampened due to the smaller substitution effect. Furthermore, for large enough wealth levels, the wealth effect becomes dominant and drives the decrease in consumption given the decrease in the interest rate.

To see how the heterogeneous pass-through affects the substitution effect, consider the households' consumption response to a change in the real rate in the model with the heterogeneous pass-through:

$$\frac{\partial c_t^{\mathrm{R}}(b,z;\Psi_t)}{\partial r_t} = \frac{\partial(1+r_t b)}{\partial r_t} - \left(\frac{\partial b'(b,z;\Psi_t)}{\partial r_t} + \frac{\partial \phi(b,z;\Psi_t)}{\partial r_t}\right). \tag{4.6.1}$$

The first component, $\frac{\partial(1+r_t b)}{\partial r_t}$, refers to the wealth effect; part of the consumption change is directly proportional to the household's level of wealth. It must be noted that this wealth effect is independent of the heterogeneous pass-through and thus *identical* in the two versions of the economy. The second component corresponds to the substitution effect; it depends on the way households adjusts their asset position, $\frac{\partial b'(b,z;\Psi_t)}{\partial r_t}$, and on the way the bank relaxes the credit conditions for households, $\frac{\partial \phi(b,z;\Psi_t)}{\partial r_t}$. Note that from Proposition 4.6, the substitution effect can be approximately written as

$$\frac{\partial b'(b,z;\Psi_t)}{\partial r_t} + \frac{\partial \phi(b,z;\Psi_t)}{\partial r_t} \simeq \left(2 + \Gamma(b,z;\Psi_t)\right) \cdot \frac{\partial \phi(b,z;\Psi_t)}{\partial r_t}. \tag{4.6.2}$$

Recall that $\Gamma(b,z,b';\Psi_t)$ is a term that captures the "pass-through" from banks to households. In the version of the economy with the heterogeneous pass-through, households must rely on banks to adjust their asset position; banks effectively have monopolistic power over households and are able to extract some of the households' surplus generated by a credit expansion. In other words, for every \$1 decrease in a bank's cost of funds, the bank decreases the credit costs for *all* households who are effectively borrowing (i.e. $b' < b$) by less than \$1. The term $\Gamma(b,z,b';\Psi_t)$ captures the surplus the bank is able to retain for every dollar decrease in its cost of funds; that is why I refer to it as the "pass-through" term. Note that the pass-through is negative, $\Gamma(b,z,b';\Psi_t) < 0$, for all effective borrowers. In the version of the economy without the heterogenous pass-through, households can adjust their asset position at any time and need not rely on a bank. Since households can trade with each other and with the government, banks have no monopolistic power. Therefore, banks can't extract any surplus from households; they just adjust the credit conditions one-for-one with the real interest rate and it is as if $\Gamma(b, z, b'; \Psi_t) = 0$. Therefore, the effect of the "pass-through" is to dampen the substitution effect across all households in the economy, as seen from equation (4.6.2).

The second term, $\frac{\partial \phi(b,z;\Psi_t)}{\partial r_t}$, refers to the way in which banks adjust the credit conditions for households in the economy. From Section 4.4, recall that this term is mainly driven by two factors. First, it is driven by the effective consumer surplus $\widetilde{CS}_t(b, z; \Psi_t)$; a measure of the value of the banking service for a household. Second, it is driven by the effective branch surplus $\widetilde{BS}_t(b, z; \Psi_t)$; a measure of the amount of resources (surplus) the bank branch can extract from a household. In short, banks are willing to relax the credit conditions more for households who have a small valuation of the banking service (in order to make it attractive for them) and for households from whom they can extract a larger amount of surplus.

When the wealth effect is taken into account, the banking service becomes less valuable for households who are currently borrowing (i.e a positive wealth effect) as $\widetilde{CS}_t(b, z; \Psi_t)$ decreases. Additionally, these households now have more resources the branch can extract as $\widetilde{BS}_t(b, z; \Psi_t)$ increases. The converse is true for households with positive wealth (i.e a negative wealth effect); they have less resources for the branch to extract *and* their valuation of the service increases. That is, due to the wealth effect banks are willing to relax more the credit conditions for households with negative asset holdings. From equation (4.6.2), this adjustment in the credit conditions across households introduces heterogeneity in the substitution effect; this effect is enhanced for households with negative assets and dampened for households with positive assets.

The combination of these two terms, the "pass-through" and the adjustment in credit conditions, results in the substitution effect being dampened for households with positive wealth and enhanced for households with negative wealth. Recall, from Section 4.5.2, that the FICO score distribution in the model is positively correlated with the level of asset holdings. Note first that households with a FICO score below 650 are those who have negative wealth holdings (see Figure 4.11). For these households, the elasticity of consumption is negative in both versions of the economy as seen in Figure 4.14; a decrease in the real interest rate (monetary expansion) increases consumption. However, the elasticity of substitution is more negative when the heterogeneous bank pass-through is included due to the enhanced substitution effect. Second, households with a FICO score above 650 are those who have positive wealth holdings; the substitution effect is dampened. Figure 4.14 shows how for these households the consumption elasticity is "less" negative when the heterogeneous pass-through is included. Furthermore, for large enough FICO score (i.e. wealth level), the wealth effect dominates and the consumption response becomes positive for the model with the heterogenous pass-through.

Overall, my model shows that the heterogeneous bank pass-through does affect the transmission mechanism of monetary policy; it enhances the transmission for low wealth and income households while dampening it for high wealth and income ones. The effect of the heterogenous pass-through on monetary policy transmission to aggregate consumption depends on the distribution of households across the economy and their consumption share. Given my model's calibration, the consumption share of high wealth and income households is dominant; the heterogenous bank pass-through decreases the aggregate consumption response to a monetary stimulus by about 5 times, relative to the "direct" aggregate consumption response in the one-asset HANK framework.

4.7 Conclusion

I show that the way in which banks change credit conditions across households after a change in their cost of funds has important implications for the transmission of monetary policy. My paper incorporates this heterogeneous bank pass-through channel via a financial friction in a New Keynesian continuous-time model that features idiosyncratic income uncertainty and incomplete markets. I show that the model's prediction about the degree of heterogeneity across households' borrowing adjustments arising from the bank pass-through is in line with some recent empirical evidence; following a change in banks' cost of funds, households at the top of the FICO score distribution adjust their borrowing by about twice as much relative to households at the bottom of the FICO score distribution.

The heterogenous bank pass-through mechanism arising from the model hinges on two factors. First, it hinges on the bank's ability to extract surplus from households; a consequence of households relying on the bank to make any asset adjustments. This is the "pass-through" part of the mechanism; for every unit decrease in the bank's cost of funds, the bank decreases the credit costs for households but by less than a unit. Second, it hinges on the fact that the bank's ability to extract surplus from households depends on the specific characteristics of the latter. This is the heterogenous part of the mechanism. In order to extract surplus from households, the bank adjusts its credit conditions taking into account two points. On one hand, the household's available resources; the bank is willing to relax the credit conditions more for households with a large amount of resources as it can extract more surplus from them. On the other hand, the household's valuation of the banking service; the bank must relax the credit conditions more for households with a relatively small valuation of the service in order to make the service more attractive for them.

When the heterogeneous bank pass-through is explicitly included, the aggregate consumption response to a change in the real rate is about five times smaller compared to the aggregate consumption response in the standard one-asset Heterogeneous Agent New Keynesian (HANK) model. This difference is mostly driven by the consumption response of wealthy households. In the HANK framework, the consumption response across households is characterized by two observations. First, consumption responds non-negatively for all households; a decrease in the real rate leads to an increase in consumption. Second, the consumption response is larger for households with smaller wealth and income levels. When the heterogeneous bank pass-through is included, only the latter observation holds. For households with sufficiently large wealth levels, a decrease in the real rate leads to a decrease in consumption. The intuition behind this results is as follows. Given the decrease in the real rate, the wealth effect implies that households with positive asset holdings are "effectively" poorer and value the banking service more. Given the heterogenous bank pass-through mechanism, banks end up relaxing the credit conditions by less for these households; which exacerbates the negative impact of the wealth effect on consumption. For households with large enough wealth levels, the wealth effect dominates and the decrease in the real rate leads to a decrease in consumption.

My results contribute to the literature that incorporates idiosyncratic uncertainty, incomplete markets, and financial frictions to study the transmission of monetary policy. My findings reaffirm two observations stressed by this literature. First, financial frictions are important for policy transmission. Second, there is a highly heterogeneous impact of monetary policy across households. Importantly, I show that even if financial frictions enhance the monetary policy transmission for some households and dampen it for some others, these effects do not undo each other on the aggregate.

Appendices

A.1 Appendix To Chapter 1

A.1.1 Proof of Proposition 1.1

Let $R_d \equiv R_0 - (1-\delta)\,\pi_{d,1} \geq 0$, $W^a_{-1} \equiv q_0\,(1-\delta)\,d^a_{-1} - R_{-1}b^a_{-1}\pi^{-1}_{c,0}$, $A_{c,t} \equiv (1-\Delta_{c,t}\omega_{c,t})$, $A_{d,t} \equiv (q_t - \Delta_{d,t}\omega_{c,t})$, and $P_1 \equiv \beta_b\,(1+(1-\alpha)\,\kappa_b)$. Since the FOC's are necessary and sufficient, the solution for the no constraint case is characterized by the following equations:

$$c^a_0 = \alpha\,(1-\alpha)^{-1}\,q_0 R_0^{-1} R_d d^a_0 \tag{A.1.1}$$

$$c^a_1 = \beta_a \alpha\,(1-\alpha)^{-1}\,q_0 \pi^{-1}_{c,1} R_d d^a_0 \tag{A.1.2}$$

$$n^a_0 = \omega_{c,0}\nu_a^{-1}\,(1-\alpha)\,R_0\,(q_0 R_d d^a_0)^{-1} \tag{A.1.3}$$

$$\frac{b^a_0 R_0}{\pi_{c,1}} = q_1\left[(1-\delta)\,d^a_0 - d^a_1\right] - c^a_1 + \omega_{c,1} n^a_1 + \Pi^a_1 \tag{A.1.4}$$

$$n^a_1 = \omega_{c,1}\,(\nu_a\beta_a)^{-1}\,(1-\alpha)\,\pi_{c,1}\,(q_0 R_d d^a_0)^{-1} \tag{A.1.5}$$

$$d^a_1 = \beta_a\,(1+\kappa_a)\,\pi^{-1}_{d,1} R_d d^a_0 \tag{A.1.6}$$

$$b^a_0 = c^a_0 + q_0 d^a_0 - \omega_{c,0} n^b_0 - W^a_{-1} - \Pi^a_0 \tag{A.1.7}$$

Note that Π^a_t denote the profits from firm ownership; thus for borrowers $\Pi^b_t = 0$ while for savers $\Pi^s_t = \Omega_s^{-1}\,(Y_{c,t}A_{c,t} + Y_{d,t}A_{d,t})$. For an agent of type borrower, using equations (A.1.1), (A.1.2), (A.1.3), (A.1.5), and (A.1.6) in the consolidated budget constraint, one obtains a second degree polynomial in c^b_0. Then $c^{\text{NC}}_{b,0}$ is a solution to the borrower's problem iff it satisfies

$$\alpha^{-1}\,(1-\alpha)\left[(1+\beta_b)\,(1-\alpha)^{-1} + \kappa_b\beta_b\right]\left(c^b_0\right)^2 - W^b_{-1}\,c^b_0 - \alpha\nu_b^{-1}\left[\omega^2_{c,0} + \beta_b^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\right] = 0. \tag{A.1.8}$$

Note that equation (A.1.8) has two solutions, denote them x_1 and x_2.
Given $\alpha \in (0,1)$, $(1+\beta_b)\,(1-\alpha)^{-1} + \kappa_b\beta_b > 0$ and $\alpha\nu_b^{-1}\left[\omega^2_{c,0} + \beta_b^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\right] \geq 0 \Longrightarrow$ $x_1, x_2 \in \mathbb{R}$, with $x_1 \leq 0 \leq x_2$. Therefore, given $R_d \geq 0$ and $R_0, q_0 > 0$, there is a unique solution $c^{\text{NC}}_{b,0} \in \mathbb{R}^+$ that satisfies (A.1.8) and it is given by

$$c^{\text{NC}}_{b,0} = \alpha\,2^{-1}\,(P_1+1)^{-1}\left[W^b_{-1} + \sqrt{\left(W^b_{-1}\right)^2 + 4\,(P_1+1)\,\nu_b^{-1}\left(\omega^2_{c,0} + \beta_b^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\right)}\right] \tag{A.1.9}$$

The remaining variables for the optimal allocation for the borrower are given by (A.1.1)-(A.1.4). Trivially, this allocation is unique given the unique value of $c^{\text{NC}}_{b,0}$. Now, given an allocation for the borrower, $\left(c^b_0, d^b_0, c^b_1\right)$, define for the saver $\overline{W}^s_{-1} \equiv W^s_{-1} - (1-\delta)\,A_{d,0}\left(d^s_{-1} + \Omega_b\Omega_s^{-1}d^b_{-1}\right)$, $\overline{A}^b_{d_0} \equiv \left[A_{d,0} - R_0^{-1}\pi_{c,1}\,(1-\delta)\,A_{d,1}\right]$, $\overline{A}^b_{c_1} \equiv R_0^{-1}\pi_{c,1}\left[A_{c,1} + (\alpha\,q_1)^{-1}\,(1-\alpha)\,(1+\kappa_b)\,A_{d,1}\right]$,

$K_{d_0}^s \equiv \left[\Delta_{d,0}\omega_{c,0} - R_0^{-1}\pi_{c,1}\left(1-\delta\right)\Delta_{d,1}\omega_{c,1}\right]$,
$K_{c_1}^s \equiv \omega_{c,1}\left[\Delta_{c,1} + \left(\alpha\, q_1\right)^{-1}\left(1-\alpha\right)\left(1+\kappa_s\right)\Delta_{d,1}\right]$, $W_F^s \equiv \Omega_b\Omega_s^{-1}\left(c_0^b A_{c,0} + d_0^b \overline{A}_{d_0}^b + c_1^b \overline{A}_{c_1}^b\right)$,
and
$K^s \equiv \Delta_{c,0}\omega_{c,0} + \beta_s K_{c_1}^s + \left(\alpha\, q_0\right)^{-1}\left(1-\alpha\right)R_0\left[R_0 - \left(1-\delta\right)\pi_{d,1}\right]^{-1}K_{d_0}^s$. Using equations (A.1.1), (A.1.2), (A.1.3), (A.1.5), and (A.1.6) in the lifetime budget constraint, one obtains a second degree polynomial in c_0^s. Then $c_{s,0}^{\mathrm{NC}}$ is a solution to the saver's problem iff it satisfies

$$K^s\left(c_0^s\right)^2 - \left(\overline{W}_{-1}^s + W_F^s\right)\, c_0^s - \alpha\nu_s^{-1}\left[\omega_{c,0}^2 + \beta_s^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\right] = 0. \tag{A.1.10}$$

As it was the case for the borrower, this equation has two solutions, denote them x_1 and x_2. Provided $K^s \geq 0 \Longrightarrow x_1, x_2 \in \mathbb{R}$, with $x_1 \leq 0 \leq x_2$. Therefore, given $R_d \geq 0$ and $R_0, q_0 > 0$, there is a unique solution $c_{s,0}^{\mathrm{NC}} \in \mathbb{R}^+$ that satisfies (A.1.10) and it is given by

$$c_{s,0}^{\mathrm{NC}} = \left(2K^s\right)^{-1}\left[\left(\overline{W}_{-1}^s + W_F^s\right) + \sqrt{\left(\overline{W}_{-1}^s + W_F^s\right)^2 + 4\alpha K^s\nu_s^{-1}\left(\omega_{c,0}^2 + \beta_s^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\right)}\right] \tag{A.1.11}$$

The remaining variables for the optimal allocation for the saver are given by (A.1.1)-(A.1.4). Trivially, this allocation is unique given the unique value of $c_{s,0}^{\mathrm{NC}}$.

A.1.2 Proof of Proposition 1.2

Define R_d, P_1, and W_{-1}^a as in the proof of Proposition 1.1, $R_d^\chi \equiv q_0R_0^{-1}\left[R_0 - \left(1-\chi\right)\left(1-\delta\right)\pi_{d,1}\right]$, $\tilde{R}_d \equiv q_0R_0^{-1}R_d$, $A_1 \equiv \chi^{-1}\left(R_0\left(1-\delta\right)^{-1}\pi_{d,1}^{-1} - 1\right)$, $A_2 \equiv \beta_b^{-1}\left(\omega_{c,1}\pi_{c,1}R_0^{-1}\right)^2\left(1+A_1\right)^2$, and $A_3 \equiv 4\,\nu_b^{-1}\left(\omega_{c,0}^2 + A_2\right)\left(1+P_1\right)$.

Note that since I assume that the agents of type saver face no constraint, given an allocation $\left(c_0^b, d_0^b, c_1^b\right)$, the proof for existence an uniqueness of the solution for the saver's problem is exactly the same as in Proposition 1.1. The same is true for the borrower's problem in an economy with non-binding credit constraints. Hence, in what follows, I consider only the problem for agents of type borrower and I treat the constraint as binding.

Let $R_d \geq 0$. Since $\chi \in (0,1) \Longrightarrow R_d^\chi \geq 0$. The necessary and sufficient FOC's for the borrower's problem are given by:

$$R_0 b_0^b = (1-\chi)(1-\delta) q_0 \pi_{d,1} d_0^b \tag{A.1.12}$$

$$\alpha \tilde{R}_d d_0^b = c_0^b \left(1 - \alpha + \chi (1-\delta) q_0 \pi_{d,1} \zeta_0 d_0^b\right) \tag{A.1.13}$$

$$\alpha c_0^b = \pi_{c,1} c_1^b \beta_b^{-1} \left(\alpha R_0^{-1} - \zeta_0 c_0^b\right) \tag{A.1.14}$$

$$n_t^b = \alpha \omega_{c,t} \left(\nu_b c_t^b\right)^{-1}, \text{ for } t \in \{0,1\} \tag{A.1.15}$$

$$d_1^b = (1-\alpha)(1+\kappa_b)(\alpha q_1)^{-1} c_1^b \tag{A.1.16}$$

$$q_0 d_0^b - b_0^b = W_{-1} + \omega_{c,0} n_0^b - c_0^b \tag{A.1.17}$$

$$\frac{b_0^b R_0}{\pi_{c,1}} = q_1 \left[(1-\delta) d_0^b - d_1^b\right] - c_1^b + \omega_{c,1} n_1^b; \tag{A.1.18}$$

where $\zeta_0 > 0$ denotes the multiplier for the credit constraint. Define the functions

$$f_1\left(c_0^b\right) = \omega_{c,0}^2 \alpha \nu_b^{-1} + W_{-1}^b c_0^b - \left(c_0^b\right)^2 \tag{A.1.19}$$

$$f_3\left(c_0^b\right) = \omega_{c,0}^2 \alpha \nu_b^{-1} + W_{-1}^b c_0^b - \alpha^{-1}\left(c_0^b\right)^2 \tag{A.1.20}$$

$$f_2\left(c_0^b\right) = \frac{\alpha \omega_{c,0}^2}{\nu_b} + W_{-1}^b c_0^b - \frac{(P_1+1)\left(c_0^b\right)^2}{\alpha} \tag{A.1.21}$$

$$g\left(c_0^b\right) = [\chi(1-\delta) q_1]^2 \left[f_1\left(c_0^b\right)\right]^2 f_2\left(c_0^b\right) + \alpha (\beta_b \nu_b)^{-1} \left(\omega_{c,1} R_d^\chi\right)^2 \left[f_3\left(c_0^b\right)\right]^2. \tag{A.1.22}$$

The system of equations (A.1.12)-(A.1.18) can be simplified to a single equation in c_0^b, which is given by $g\left(c_0^b\right) = 0$. In addition, using the definitions (A.1.19)-(A.1.21), d_0^b, c_1^b and ζ_0 can be written as functions of c_0^b:

$$c_1^b\left(c_0^b\right) = \beta_b \chi (1-\delta) q_1 c_0^b \left[R_d^\chi \cdot f_3\left(c_0^b\right)\right]^{-1} \cdot \left[f_1\left(c_0^b\right)\right] \tag{A.1.23}$$

$$\zeta_0\left(c_0^b\right) = \alpha \left(R_0 c_0^b\right)^{-1} \left[1 - (1+A_1) f_3\left(c_0^b\right) f_1^{-1}\left(c_0^b\right)\right] \tag{A.1.24}$$

$$d_0^b\left(c_0^b\right) = \left[f_1\left(c_0^b\right)\right] \cdot \left(R_d^\chi c_0^b\right)^{-1} \tag{A.1.25}$$

The remaining allocation variables $\left(b_0^b, n_0^b, n_1^b, d_1^b\right)$ are given by (A.1.12), (A.1.15) and (A.1.16), respectively. Hence $c_{b,0}^C$ is a solution to the borrower's problem iff $c_{b,0}^C, f_1\left(c_{b,0}^C\right), f_3\left(c_{b,0}^C\right) \geq 0$ and $g\left(c_{b,0}^C\right) = 0$.

First, from (A.1.19), (A.1.20) and $\alpha \in (0,1)$, $f_1 \geq f_3 \ \forall c_0^b \geq 0$. Second, let $x_1^{f_3}$ and $x_2^{f_3}$ denote the roots of f_3. Since $f_3(0) = \omega_{c,0}^2 \alpha \nu_b^{-1} \geq 0$ and $f_3'' = -2\alpha^{-1} < 0 \ \forall c_0^b \geq 0 \Longrightarrow x_1^{f_3}, x_2^{f_3} \in \mathbb{R}^+$, with $x_1^{f_3} \leq 0 \leq x_2^{f_3}$. Hence $f_3\left(c_0^b\right) < 0 \ \forall c_0^b \in \left(-\infty, x_1^{f_3}\right) \cup \left(x_2^{f_3}, +\infty\right)$ and $f_3\left(c_0^b\right) \geq 0$ $\forall c_0^b \in \left[x_1^{f_3}, x_2^{f_3}\right]$. Hence $c_{b,0}^C, f_1\left(c_{b,0}^C\right), f_3\left(c_{b,0}^C\right) \geq 0 \Longleftrightarrow c_{b,0}^C \in \left[0, x_2^{f_3}\right]$.

In addition, if $g\left(c_{b,0}^C\right) = 0 \Longrightarrow f_2\left(c_{b,0}^C\right) < 0$. Let $x_1^{f_2}$ and $x_2^{f_2}$ denote the roots of f_2. Since $f_2(0) = \omega_{c,0}^2 \alpha \nu_b^{-1} \geq 0$ and $f_2'' = -2\alpha^{-1}(P_1+1) < 0 \ \forall c_0^b \geq 0 \Longrightarrow x_1^{f_2}, x_2^{f_2} \in \mathbb{R}^+$, with $x_1^{f_2} \leq 0 \leq x_2^{f_2}$. Hence $f_2\left(c_0^b\right) \leq 0 \ \forall c_0^b \in \left(-\infty, x_1^{f_2}\right] \cup \left[x_2^{f_2}, +\infty\right)$ and $f_2\left(c_0^b\right) > 0 \ \forall c_0^b \in \left(x_1^{f_2}, x_2^{f_2}\right)$.

Therefore, $g\left(c_{b,0}^{C}\right)=0\,,\,c_{b,0}^{C}\in\left[0,x_2^{f_3}\right]\Longleftrightarrow g\left(c_{b,0}^{C}\right)=0\,,\,c_{b,0}^{C}\in\left[x_2^{f_2},x_2^{f_3}\right]$; where

$$x_2^{f_2}=\alpha\left[2\left(P_1+1\right)\right]^{-1}\left[W_{-1}^{b}+\sqrt{\left(W_{-1}^{b}\right)^2+4\omega_{c,0}^2\nu_b^{-1}\left(P_1+1\right)}\right] \tag{A.1.26}$$

$$x_2^{f_3}=\alpha 2^{-1}\left[W_{-1}^{b}+\sqrt{\left(W_{-1}^{b}\right)^2+4\omega_{c,0}^2\nu_b^{-1}}\right]. \tag{A.1.27}$$

Note that $x_2^{f_2}<x_2^{f_3}$ since $P_1>0$ by definition.

Finally, $f_2\left(x_2^{f_2}\right)=f_3\left(x_2^{f_3}\right)=0\implies g\left(x_2^{f_2}\right)>0$ and $g\left(x_2^{f_3}\right)<0$. By noting that $g\left(c_0^b\right)$ is a continuous function, the Intermediate Value Theorem ensures the existence of $c_{b,0}^{C}\in\left(x_2^{f_2},x_2^{f_3}\right)$ s.t. $g\left(c_{b,0}^{C}\right)=0$.

I show next that if $P_1>1$ and $\left(W_{-1}^{b}\right)^2\geq\max\left\{A_3-8\nu_b^{-1}\omega_{c,0}^2\left(1+P_1\right),\,A_3\left(P_1^2-1\right)^{-1}\right\}$, then this solution is unique. To this end, let

$$f_4\left(c_0^b\right)=\alpha^{-1}\left(P_1+1\right)\left(c_0^b\right)^2-W_{-1}^{b}c_0^b-\alpha\nu_b^{-1}\left(\omega_{c,0}^2+A_2\right). \tag{A.1.28}$$

Now, for $c_0^b\in\left[x_2^{f_2},x_2^{f_3}\right)$, we have $g\left(c_0^b\right)<-\left[\chi\left(1-\delta\right)q_1\right]^2\cdot\left[f_3\left(c_0^b\right)\right]^2\cdot f_4\left(c_0^b\right)$, where I have used the fact that $f_1>f_3$. So that, for $c_0^b\in\left[x_2^{f_2},x_2^{f_3}\right)$, if $f_4\left(c_0^b\right)\geq 0\implies g\left(c_0^b\right)<0$. Denote the roots of f_4 by $x_1^{f_4}$ and $x_2^{f_4}$. From (A.1.28) and since $P_1,A_2>0$, it is clear that $f_4\left(0\right)<0$ and $f_4''>0$, $\forall c_0^b\implies x_1^{f_4},x_2^{f_4}\in\mathbb{R}^{+}$, with $x_1^{f_4}\leq 0\leq x_2^{f_4}$. Therefore, $f_4\left(c_0^b\right)\geq 0\;\forall c_0^b\in\left(-\infty,x_1^{f_4}\right]\cup\left[x_2^{f_4},+\infty\right)$ and $f_4\left(c_0^b\right)<0\;\forall c_0^b\in\left(x_1^{f_4},x_2^{f_4}\right)$. It follows that if $g\left(c_{b,0}^{C}\right)=0$ with $c_{b,0}^{C}\in\left(x_2^{f_2},x_2^{f_3}\right)\implies c_{b,0}^{C}\in\left(x_2^{f_2},\min\left\{x_2^{f_3},x_2^{f_4}\right\}\right)$; where

$$x_2^{f_4}=\alpha\left[2\left(P_1+1\right)\right]^{-1}\left[W_{-1}^{b}+\sqrt{\left(W_{-1}^{b}\right)^2+4\nu_b^{-1}\left(\omega_{c,0}^2+A_2\right)\left(P_1+1\right)}\right] \tag{A.1.29}$$

Trivially $x_2^{f_2}<x_2^{f_4}$.

Given $P_1>1$, $W_{-1}^{b}\geq 0$, and $\left(W_{-1}^{b}\right)^2\geq A_3\left(P_1^2-1\right)^{-1}$
$\implies\left(W_{-1}^{b}\right)^2P_1^2\geq\left(W_{-1}^{b}\right)^2+4\nu_b^{-1}\left(\omega_{c,0}^2+A_2\right)\left(P_1+1\right)$
$\implies W_{-1}^{b}\left(P_1+1\right)\geq W_1^{b}+\sqrt{\left(W_{-1}^{b}\right)^2+4\nu_b^{-1}\left(\omega_{c,0}^2+A_2\right)\left(P_1+1\right)}\implies\alpha 2^{-1}W_{-1}^{b}\geq x_2^{f_4}\implies$ from (A.1.27), $x_2^{f_3}\geq x_2^{f_4}$. Thus $c_{b,0}^{C}\in\left(x_2^{f_2},x_2^{f_4}\right)$. Also, $x_{max}^{f_3}\equiv\alpha 2^{-1}W_{-1}^{b}$ is the value at which f_3 attains its maximum. Clearly, $x_{max}^{f_3}\in\left[0,x_2^{f_3}\right]\implies f_3'\geq 0$ for $c_0^b\in\left[0,x_{max}^{f_3}\right]\implies f_1',f_3'>0$ and $f_1,f_3>0$ for $c_0^b\in\left[x_2^{f_2},x_2^{f_4}\right)$, since, from (A.1.19) and (A.1.20), $f_1\geq f_3$ and $f_1'\geq f_3'$,

$\forall c_0^b \geq 0$. Define the function

$$f_5\left(c_0^b\right) = 2\left[\alpha\left(\beta_b\nu\right)_b^{-1}\left(\omega_{c,1}R_d^{\chi}\right)^2 + f_2\left[\chi\left(1-\delta\right)q_1\right]^2\right] + f_1\left(f_1'\right)^{-1}f_2'\left[\chi\left(1-\delta\right)q_1\right]^2, \tag{A.1.30}$$

and note that $f_5' < 0$, $\forall c_0^b \in \left[x_2^{f_2}, x_2^{f_4}\right)$ since $f_2', f_2'', f_1'' < 0$ and $f_1, f_1' > 0$ on this interval. Consider now g',

$$g'\left(c_0^b\right) = 2\left[f_3 f_3'\alpha\left(\beta_b\nu_b\right)^{-1}\left(\omega_{c,1}R_d^{\chi}\right)^2 + f_1 f_1' f_2\left[\chi\left(1-\delta\right)q_1\right]^2\right] + f_1^2 f_2'\left[\chi\left(1-\delta\right)q_1\right]^2,\ c_0^b \in \left[x_2^{f_2}, x_2^{f_4}\right]. \tag{A.1.31}$$

So that, for $c_0^b \in \left[x_2^{f_2}, x_2^{f_4}\right)$, $g' \leq f_1 f_1' f_5$. It follows that, if $f_5 < 0$ on $\left(x_2^{f_2}, x_2^{f_4}\right) \implies g' < 0$ on $\left(x_2^{f_2}, x_2^{f_4}\right)$. Thus, to show $g' < 0$ on $\left(x_2^{f_2}, x_2^{f_4}\right)$, it suffices to show $f_{5,*} \equiv f_5\left(x_2^{f_2}\right) \leq 0$. Consider

$$f_{2,*}' \equiv f_2'\left(x_2^{f_2}\right) = -\sqrt{\left(W_{-1}^b\right)^2 + 4\nu_b^{-1}\omega_{c,0}^2\left(1+P_1\right)} \tag{A.1.32}$$

$$f_{1,*}' \equiv f_1'\left(x_2^{f_2}\right) = \left(1 - \frac{\alpha}{1+P_1}\right)W_{-1}^b + \left(\frac{\alpha}{1+P_1}\right)f_{2,*}' \tag{A.1.33}$$

$$\left(f_1 \cdot f_2\right)_* \equiv f_1 \cdot f_2'\left(x_2^{f_2}\right) = -\frac{\alpha\left(W_{-1}^b\right)^2}{2\left(1+P_1\right)}\left[f_{1,*}' - f_{2,*}'\right] - \left(\frac{2\alpha\omega_{c,0}^2}{\nu_b}\right)f_{1,*}' + \frac{\alpha\omega_{c,0}^2}{\nu_b}\left[1 + \frac{\alpha}{1+P_1}\right]f_{2,*}' \tag{A.1.34}$$

$$\begin{aligned}
\implies\ f_{5,*} &= \left[\chi\left(1-\delta\right)\right]^2\left(f_{1,*}'\right)^{-1}\left[2\alpha\nu_b^{-1}A_2 f_{1,*}' + \left(f_1 f_2'\right)_*\right] \\
&\leq \left[\chi\left(1-\delta\right)\right]^2\left(f_{1,*}'\right)^{-1}\left[2\alpha\nu_b^{-1}A_2 f_{1,*}' - \frac{\alpha W_{-1}^2}{2\left(1+P_1\right)}f_{1,*}' - \left(\frac{2\alpha\omega_{c,0}^2}{\nu_b}\right)f_{1,*}'\right] \\
&= \left[\chi\left(1-\delta\right)\right]^2 2^{-1}\alpha\left(1+P_1\right)^{-1}\left[A_3 - 8\nu_b^{-1}\omega_{c,0}^2\left(1+P_1\right) - \left(W_{-1}^b\right)^2\right] \\
&\leq 0.
\end{aligned}$$

The first inequality follows since since $f_{2,*}' \leq 0$. The second inequality is a consequence of the assumption $\left(W_{-1}^b\right)^2 \geq \max\left\{A_3 - 8\nu_b^{-1}\omega_{c,0}^2\left(1+P_1\right),\ A_3\left(P_1^2-1\right)^{-1}\right\}$. Hence $g'\left(c_0^b\right) < 0$ on $\left(x_2^{f_2}, x_2^{f_4}\right)$. Since $c_{b,0}^C \geq 0$ is a solution to the borrower's problem $\iff g\left(c_{b,0}^C\right) = 0$ and $c_{b,0}^C \in \left(x_2^{f_2}, x_2^{f_4}\right)$, this solution is unique.

A.1.3 Proof of Proposition 1.3

Given $\hat{P}^{NC}$, define $W_{-1} \equiv W^b_{-1}$, P_1, and A_1 as in the proof of Proposition 1.2. Also, let $x_2^{f_2}$, $x_2^{f_4}$, $f_1\left(c_0^b\right)$, $f_3\left(c_0^b\right)$, and $\zeta_0\left(c_0^b\right)$ be given by (A.1.26), (A.1.29), (A.1.19), (A.1.20), and (A.1.24). Recall that $\zeta_0\left(c_0^b\right)$ refers to the multiplier on the credit constraint. In addition, define
$B_1 \equiv \beta_b^{-1}\left(\hat{\omega}_{c,1}^{\text{NC}}\hat{\pi}_{c,1}^{\text{NC}}\left(\hat{R}_0^{\text{NC}}\right)^{-1}\right)^2$, $B_2 \equiv 4\nu_b^{-1}\left({\hat{\omega}_{c,0}^{\text{NC}}}^2 + B_1\right)(1+P_1)$,
and $B_3 \equiv 4\nu_b^{-1}A_1\left(A_1P_1 - (1-\alpha)\right)^{-1}B_1\left(1+P_1\right)^2$.

Therefore, the credit constraint is binding for the allocation $\hat{A}_b^{NC} \iff \zeta\left(\hat{c}_{b,0}^{NC}\right) \geq 0$

$\iff f_1\left(\hat{c}_{b,0}^{NC}\right) \geq (1+A_1)\, f_3\left(\hat{c}_{b,0}^{NC}\right)$, since $\hat{c}_{b,0}^{NC} \in \left(x_2^{f_2}, x_2^{f_4}\right)$ and $f_1, f_3 \geq 0$ on this interval,

$\iff (1+A_1)\, f_3\left(\hat{c}_{b,0}^{NC}\right) - f_1\left(\hat{c}_{b,0}^{NC}\right) \leq 0$

$\iff A_1W_{-1}\hat{c}_{b,0}^{NC} + A_1\alpha\nu_b^{-1}\left(\hat{\omega}_{c,0}^{\text{NC}}\right)^2 - \alpha^{-1}\left(1+A_1-\alpha\right)\left(\hat{c}_{b,0}^{NC}\right)^2 \leq 0$

$\iff \left[A_1 - \left(1-\alpha+A_1\right)\left(1+P_1\right)^{-1}\right]\left(W_{-1} + \sqrt{W_{-1}^2 + B_2}\right)^2 \leq 4\nu_b^{-1}A_1B_1\left(1+P_1\right)$

$\iff \left(W_{-1} + \sqrt{W_{-1}^2 + B_2}\right)^2 \leq B_3$ whenever $A_1P_1 > (1-\alpha)$

$\iff W_1 \leq \left(B_3 - B_2\right)\left(2\sqrt{B_3}\right)^{-1}$ whenever $A_1P_1 > (1-\alpha)$

A.1.4 Proof of Proposition 1.4

Note that given the definition of $U(\cdot)$, one has $\left(\dfrac{\partial U}{\partial c_0}\right), \left(\dfrac{\partial U}{\partial c_1}\right), \left(\dfrac{\partial U}{\partial d_0}\right) \geq 0$ and $\left(\dfrac{\partial^2 U}{\partial {c_0}^2}\right)$, $\left(\dfrac{\partial^2 U}{\partial {c_1}^2}\right), \left(\dfrac{\partial^2 U}{\partial {d_0}^2}\right) \leq 0$.

A note on notation first. Let $x = \left\{c_t^b, d_t^b, n_t^b\right\}_{t=0}^{t=1}$. Since $U(x)$ is assumed to be additively separable in all its arguments, then $\dfrac{\partial U}{\partial x_i}$ for $x_i \in x$ is only a function of x_i. Therefore, for simplicity, let $\dfrac{\partial U}{\partial x_i}(a) \equiv \left.\dfrac{\partial U}{\partial x_i}\right|_{x_i=a}$. Also, I let $\left\{\pi_{c,t}^{\text{NC}}, \pi_{d,t}^{\text{NC}}, \omega_{c,t}^{\text{NC}}, \omega_{d,t}^{\text{NC}}, q_t^{\text{NC}}, R_0^{\text{NC}}\right\}_{t=0}^1 \in \hat{P}^{NC}$ and $\left\{c_{b,t}^{\text{NC}}, d_{b,t}^{\text{NC}}, n_{b,t}^{\text{NC}}, b_{b,0}^{\text{NC}}\right\}_{t=0}^1 \equiv \hat{A}_b^{NC}$, and omit the '$\hat{x}$' notation for convenience.

Note that given $\left(d_0^b, b_0^b\right)$, an allocation $\left(c_t^b, d_t^b, n_t^b, b_0^b\right)$ can be constructed as follows. Let

$$A_0\left(d_0^b, b_0^b\right) = b_0^b - q_0^{NC}\left[d_0^b - (1-\delta)\, d_{-1}^b\right] - R_{-1}^{NC} b_{-1}^b \left(\pi_{c,0}^{NC}\right)^{-1} \tag{A.1.35}$$

$$A_1\left(d_0^b, b_0^b\right) = q_1^{NC}(1-\delta)\, d_0^b - R_0^{NC} b_0^b \left(\pi_{c,1}^{NC}\right)^{-1} \tag{A.1.36}$$

$$c_0^b\left(d_0^b, b_0^b\right) = 2^{-1}\left[A_0\left(d_0^b, b_0^b\right) + \left(A_0^2\left(d_0^b, b_0^b\right) + 4\left(\omega_{c,0}^{NC}\right)^2 \alpha\, \nu_b^{-1}\right)^{1/2}\right] \tag{A.1.37}$$

$$n_t^b\left(d_0^b, b_0^b\right) = \omega_{c,t}^{NC}\alpha\left[\nu_b\, c_t^b\left(d_0^b, b_0^b\right)\right]^{-1} \tag{A.1.38}$$

$$d_1^b\left(d_0^b, b_0^b\right) = (1-\alpha)(1+\kappa_b)\,\alpha^{-1}\left(q_1^{NC}\right)^{-1} c_1^b\left(d_0^b, b_0^b\right) \tag{A.1.39}$$

$$\begin{aligned} c_1^b\left(d_0^b, b_0^b\right) &= 2^{-1}\left[1+\kappa_b(1-\alpha)\right]^{-1} \cdot \\ &\quad \left[A_1\left(d_0^b, b_0^b\right) + \left(A_1^2\left(d_0^b, b_0^b\right) + 4\left[1+\kappa_b(1-\alpha)\right]\left(\omega_{c,1}^{NC}\right)^2 \nu_b^{-1}\right)^{1/2}\right] \end{aligned} \tag{A.1.40}$$

Where (A.1.35) and (A.1.36) refer to the total non-labor income of an agent of type borrower in periods 0 and 1, respectively. The non-durable consumption for each period, expressions (A.1.37) and (A.1.40), are obtained using the budget constraints and the labor FOC's. The corresponding labor supplied is given by (A.1.38), where $t \in \{0, 1\}$. Finally, the durable good consumption for period 1, expression (A.1.39), is obtained via the FOC for durable consumption for period 1. Note that for any allocation for which the constraint is binding we have $b_0^b = (1-\chi)(1-\delta)\, q_0^{NC}\pi_{d,1}^{NC}\left(R_0^{NC}\right)^{-1} d_0^b$. Hence expressions (A.1.35) - (A.1.39) are functions of d_0^b only. In particular $c_0^b\left(d_0^b\right)$, $c_1^b\left(d_0^b\right)$ and

$$\frac{\partial c_0^b}{\partial d_0^b} = -c_0^b\, q_0^{NC}\left(1 - \Gamma_{\chi,\delta}\,\pi_{d,1}^{NC}\left(R_0^{NC}\right)^{-1}\right)\left(A_0^2\left(d_0^b\right) + 4\left(\omega_{c,0}^{NC}\right)^2 \alpha\,\nu_b^{-1}\right)^{-1/2} \tag{A.1.41}$$

$$\frac{\partial c_1^b}{\partial d_0^b} = \chi(1-\delta)\, c_1^b\, q_1^{NC}\left(A_1^2\left(d_0^b\right) + 4\left[1+\kappa_b(1-\alpha)\right]\left(\omega_{c,1}^{NC}\right)^2 \nu_b^{-1}\right)^{-1/2} \tag{A.1.42}$$

where $\Gamma_{\chi,\delta} \equiv (1-\chi)(1-\delta)$. Given this, for an allocation in which the constraint is binding, define the function

$$F\left(d_0^b\right) = \left(\frac{\partial U}{\partial c_0^b}\right)\frac{q_0^{NC}}{R_0^{NC}}\left(R_0^{NC} - \Gamma_{\chi,\delta}\,\pi_{d,1}^{NC}\right) - \left(\frac{\partial U}{\partial c_1^b}\right)\chi(1-\delta)\cdot q_1^{NC} - \frac{\partial U}{\partial d_0^b}. \tag{A.1.43}$$

Then,

$$\frac{\partial F}{\partial d_0^b} = \left(\frac{\partial^2 U}{\partial {c_0^b}^2}\right)\left(\frac{\partial c_0^b}{\partial d_0^b}\right)\frac{q_0^{NC}}{R_0^{NC}}\Big(R_0^{NC} - \Gamma_{\chi,\delta}\,\pi_{d,1}^{NC}\Big) - \left(\frac{\partial^2 U}{\partial {c_1^b}^2}\right)\left(\frac{\partial c_1^b}{\partial d_0^b}\right)\chi\,(1-\delta)\cdot q_1^{NC} - \frac{\partial^2 U}{\partial {d_0^b}^2}. \tag{A.1.44}$$

For the binding constraint case and given the no constraint prices, the choice of d_0^b, and its corresponding allocation constructed using (A.1.35)-(A.1.39), is optimal iff $F\left(d_0^b\right) = 0$. This follows since $F\left(d_0^b\right) = 0$ is the FOC with respect to durable consumption in period 0. Given the assumptions on U, the FOC's are necessary and sufficient to characterize the solution. Furthermore, $\frac{\partial c_1^b}{\partial d_0^b} \geq 0$ from (A.1.42) and given $\left(\frac{\partial^2 U}{\partial {c_0}^2}\right), \left(\frac{\partial^2 U}{\partial {c_1}^2}\right), \left(\frac{\partial^2 U}{\partial {d_0}^2}\right) \leq 0$, from (A.1.44) we have

$$\text{If } \left(R_0^{NC} - \Gamma_{\chi,\delta}\,\pi_{d,1}^{NC}\right) \geq 0 \Longrightarrow \frac{\partial c_0^b}{\partial d_0^b} \leq 0 \text{ from (A.1.41)} \Longrightarrow \frac{\partial F}{\partial d_0^b} \geq 0. \tag{A.1.45}$$

Next, construct an allocation for the borrower $\left(c_{b,t}^I, d_{b,t}^I, n_{b,t}^I, b_{b,0}^I\right)$ by setting $d_{b,0}^I = d_{b,0}^{NC}$ and $b_{b,0}^I = (1-\chi)(1-\delta)\,q_0^{NC}\pi_{d,1}^{NC}\left(R_0^{NC}\right)^{-1} d_{b,0}^I$, which corresponds to letting the borrower hold the maximum allowed level of debt. Then $A_t^I \equiv A_t\left(d_{c,0}^I, b_{b,0}^I\right)$, $c_{b,t}^I \equiv c_t^b\left(d_{c,0}^I, b_{b,0}^I\right)$, $n_{b,t}^I \equiv n_t^b\left(d_{c,0}^I, b_{b,0}^I\right)$, and $d_{b,1}^I \equiv d_1^b\left(d_{c,0}^I, b_{b,0}^I\right)$. Similarly, for the allocation $\left(c_{b,t}^{NC}, d_{b,t}^{NC}, n_{b,t}^{NC}, b_{b,0}^{NC}\right)$ we have $A_t^{NC} \equiv A_t\left(d_{c,0}^{NC}, b_{b,0}^{NC}\right)$, $c_{b,t}^{NC} = c_t^b\left(d_{c,0}^{NC}, b_{b,0}^{NC}\right)$, $n_{b,t}^{NC} = n_t^b\left(d_{c,0}^{NC}, b_{b,0}^{NC}\right)$, and $d_{b,1}^{NC} = d_1^b\left(d_{c,0}^{NC}, b_{b,0}^{NC}\right)$.

Given the no constraint prices, the allocation $\left(c_{b,t}^{NC}, d_{b,t}^{NC}, n_{b,t}^{NC}, b_{b,0}^{NC}\right)$ satisfies the FOC's for durable consumption and debt holdings for the no constraint case,

$$\text{FOC debt:}\quad \frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^{NC}\Big) - \frac{\partial U}{\partial c_1^b}\Big(c_{b,1}^{NC}\Big)\cdot R_0^{NC}\cdot\left(\pi_{c,1}^{NC}\right)^{-1} = 0 \tag{A.1.46}$$

$$\text{FOC durable:}\quad \frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^{NC}\Big)\cdot q_0^{NC} - \frac{\partial U}{\partial c_1^b}\Big(c_{b,1}^{NC}\Big)\cdot(1-\delta)\cdot q_1^{NC} - \frac{\partial U}{\partial d_0^b}\Big(d_{b,0}^{NC}\Big) = 0. \tag{A.1.47}$$

Now, consider the allocation $\left(c_{b,t}^I, d_{b,t}^I, n_{b,t}^I, b_{b,0}^I\right)$. Using (A.1.46) and (A.1.47), one can write $F\left(d_{b,0}^I\right)$ as follows:

$$
\begin{aligned}
F\left(d_{b,0}^I\right) = & \left[\frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^I\Big) - \frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^{NC}\Big)\right] \frac{q_0^{NC}}{R_0^{NC}}\Big(R_0^{NC} - (1-\delta)\,\pi_{d,1}^{NC}\Big) \; + \\
& \chi\,(1-\delta)\,q_1^{NC}\left\{\frac{\pi_{c,1}^{NC}}{R_0^{NC}}\left[\frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^I\Big) - \frac{\partial U}{\partial c_0^b}\Big(c_{b,0}^{NC}\Big)\right] - \left[\frac{\partial U}{\partial c_1^b}\Big(c_{b,1}^I\Big) - \frac{\partial U}{\partial c_1^b}\Big(c_{b,1}^{NC}\Big)\right]\right\} \\
\geq & \; 0. \qquad \text{(A.1.48)}
\end{aligned}
$$

To see why we have the inequality, note that $\left(\dfrac{\partial U}{\partial c_0}\right), \left(\dfrac{\partial U}{\partial d_0}\right) \geq 0$ along with (A.1.46) and (A.1.47) imply that

$\Big(R_0^{NC} - (1-\delta)\,\pi_{d,1}^{NC}\Big) \geq 0 \Longrightarrow$ from (A.1.45), we have that $\dfrac{\partial F}{\partial d_0^b} \geq 0$. Furthermore, by definition $b_{b,0}^I \leq b_{b,0}^{NC}$ and $d_{b,0}^I = d_{b,0}^{NC} \Longrightarrow A_0^I \leq A_0^{NC}$ from (A.1.35) and $A_1^I \geq A_1^{NC}$ from (A.1.36) $\Longrightarrow c_{b,0}^I \leq c_{b,0}^{NC}$ from (A.1.37) and $c_{b,1}^I \geq c_{b,1}^{NC}$ from (A.1.40). So that $\left(\dfrac{\partial^2 U}{\partial {c_0}^2}\right), \left(\dfrac{\partial^2 U}{\partial {c_1}^2}\right) \leq 0$ imply that $\left[\dfrac{\partial U}{\partial c_0^b}\Big(c_{b,0}^I\Big) - \dfrac{\partial U}{\partial c_0^b}\Big(c_{b,0}^{NC}\Big)\right] \geq 0$ and $\left[\dfrac{\partial U}{\partial c_1^b}\Big(c_{b,1}^I\Big) - \dfrac{\partial U}{\partial c_1^b}\Big(c_{b,1}^{NC}\Big)\right] \leq 0$. Hence (A.1.48) follows.

The allocation $\left(\tilde{c}_t^b, \tilde{d}_t^b, \tilde{n}_t^b, \tilde{b}_0^b\right)$ is optimal given the no constraint prices, so that from (A.1.48) we have

$F\left(d_{b,0}^I\right) \geq F\left(\tilde{d}_0^b\right) = 0$. Since $\dfrac{\partial F}{\partial d_0^b} \geq 0 \Longrightarrow d_{b,0}^I = d_{b,0}^{NC} \geq \tilde{d}_0^b$. Given the binding constraint, it follows that

$\tilde{b}_0^b = (1-\chi)\,(1-\delta)\,q_0^{NC}\pi_{d,1}^{NC}\left(R_0^{NC}\right)^{-1}\tilde{d}_0^b \leq (1-\chi)\,(1-\delta)\,q_0^{NC}\pi_{d,1}^{NC}\left(R_0^{NC}\right)^{-1}d_{b,0}^{NC} < b_{b,0}^{NC}$.

This concludes the proof of part a).

For part b), given $\hat{P}^{NC}$, define $x_2^{f_2}, x_2^{f_4}, g\left(c_0^b\right), f_1\left(c_0^b\right), f_2\left(c_0^b\right), f_3\left(c_0^b\right)$ as in (A.1.26), (A.1.29), (A.1.22), (A.1.19), (A.1.21) and (A.1.20); respectively. In addition, let P_1, A_1, B_1, B_2, and B_3 be given as in the proof of Proposition 1.3. Suppose that the assumptions of Proposition 1.2 hold and note that, since $\hat{A}_b^{NC}$ is an allocation for which the constraint binds, we also have $W_1 \leq (B_3 - B_2)\left(2\sqrt{B_3}\right)^{-1}$ whenever $A_1 P_1 > (1-\alpha)$.

Then $\tilde{c}_0^b \in \left(x_2^{f_2}, x_2^{f_4}\right)$ is unique and must satisfy $g\left(\tilde{c}_0^b\right) = 0$. Furthermore, $f_1, f_3 > 0$, $g' < 0$ on this interval. Hence, to show that $\tilde{c}_0^b \leq c_{b,0}^{NC}$ it suffices to show $g\left(c_{b,0}^{NC}\right) \leq 0$. From Proposition 1.1, $c_{b,0}^{NC}$ is given by (A.1.1) and must satisfy (A.1.8) $\Longrightarrow f_2\left(c_{b,0}^{NC}\right) = -\alpha\nu_b^{-1}B_1 \Longrightarrow g\left(c_{b,0}^{NC}\right) = \alpha\left(\beta_b\nu_b\right)^{-1} B_4\, h\left(c_{b,0}^{NC}\right)$, where $h\left(c_{b,0}^{NC}\right) = (1+A_1)^2\left[f_3\left(c_{b,0}^{NC}\right)\right]^2 - \left[f_1\left(c_{b,0}^{NC}\right)\right]^2$ and $B_4 \equiv \left(\omega_{c,1}^{\mathrm{NC}} q_0^{\mathrm{NC}}\left(R_0^{\mathrm{NC}}\right)^{-1}\right)^2\left[\chi(1-\delta)\,\pi_{d,1}^{\mathrm{NC}}\right]^2$.

Therefore, if $h\left(c_{b,0}^{NC}\right) \leq 0 \Longrightarrow g\left(c_{b,0}^{NC}\right) \leq 0$. Now, $h\left(c_{b,0}^{NC}\right) \leq 0 \Longleftrightarrow (1+A_1)\, f_3\left(c_{b,0}^{NC}\right) \leq f_1\left(c_{b,0}^{NC}\right)$, since $c_{b,0}^{NC} \in \left(x_2^{f_2}, x_2^{f_4}\right)$ and $f_1, f_3 \geq 0$ on this interval. Now

$$
\begin{aligned}
& (1+A_1)\, f_3\left(c_{b,0}^{NC}\right) - f_1\left(c_{b,0}^{NC}\right) \leq 0 \\
\Longleftrightarrow\ & A_1 W_{-1} c_{b,0}^{NC} + A_1\alpha\nu_b^{-1}\left(\omega_{c,0}^{\mathrm{NC}}\right)^2 - \alpha^{-1}(1+A_1-\alpha)\left(c_{b,0}^{NC}\right)^2 \leq 0 \\
\Longleftrightarrow\ & \left[A_1 - (1-\alpha+A_1)(1+P_1)^{-1}\right]\left(W_{-1} + \sqrt{W_{-1}^2 + B_2}\right)^2 \leq 4\nu_b^{-1} A_1 B_1 (1+P_1)
\end{aligned}
\tag{A.1.49}
$$

Note that since $P_1, A_1, B_1 \geq 0$, then (A.1.49) is trivially satisfied whenever $A_1 P_1 \leq (1-\alpha)$. If $A_1 P_1 > (1-\alpha)$, then (A.1.49) is satisfied $\Longleftrightarrow \left(W_{-1} + \sqrt{W_{-1}^2 + B_2}\right)^2 \leq B_3 \Longleftrightarrow W_1 \leq (B_3 - B_2)\left(2\sqrt{B_3}\right)^{-1}$.

This shows that $h\left(c_{b,0}^{NC}\right) \leq 0 \Longrightarrow g\left(c_{b,0}^{NC}\right) \leq g\left(\tilde{c}_0^b\right) = 0 \Longrightarrow c_{b,0}^{NC} \geq \tilde{c}_0^b$. From (A.1.38) it follows immediately that $\tilde{n}_0^b \geq n_{b,0}^{\mathrm{NC}}$. This concludes the proof of part b).

For part c), note that $c_{b,0}^{NC} \geq \tilde{c}_0^b \Longleftrightarrow A_0^{NC} \geq \tilde{A}_0 \Longleftrightarrow b_{b,0}^{NC} - \tilde{b}_0^b \geq q_0^{\mathrm{NC}}\left(d_{b,0}^{NC} - \tilde{d}_0^b\right) \geq 0$; where the first equivalence follows from (A.1.37) and part b), the second from (A.1.35), and the last inequality from part a). Using (A.1.36), we can write

$A_1^{NC} - \tilde{A}_1 = -\left(b_{b,0}^{NC} - \tilde{b}_0^b\right)\left(\pi_{c,1}^{\mathrm{NC}}\right)^{-1}\left(R_0^{\mathrm{NC}} - q_0^{\mathrm{NC}}\left(d_{b,0}^{NC} - \tilde{d}_0^b\right)\left(b_{b,0}^{NC} - \tilde{b}_0^b\right)^{-1}(1-\delta)\,\pi_{d,1}^{\mathrm{NC}}\right) \leq 0.$

Finally, from (A.1.40) we have $A_1^{NC} \leq \tilde{A}_1 \Longleftrightarrow c_{b,1}^{NC} \leq \tilde{c}_1^b$. To conclude the proof of part c), note that (A.1.38) and (A.1.39) imply $n_{b,1}^{NC} \geq \tilde{n}_1^b$ and $d_{b,1}^{NC} \leq \tilde{d}_1^b$, respectively.

A.1.5 Proof of Proposition 1.5

Given the prices P^E, let the allocation A_a^E satisfy the budget constraint for $a \in \{s, b\}$ and $t \in \{0, 1\}$. In addition, suppose that $\Omega_b R_{-1} b_{-1}^b + \Omega_s R_{-1} b_{-1}^s = 0$. Note that the profits derived from firm ownership are given by $\Pi_t^s = \Omega_s^{-1}\left[(Y_{c,t} - \omega_{c,t} N_{c,t}) + (q_t Y_{d,t} - \omega_{c,t} N_{d,t})\right]$ for an agent of type saver and $\Pi_t^b = 0$ for an agent of type borrower. Finally, for notational convenience, let BC_t^a denote the budget constraint of agent of type a in period t.

Note that $\Omega_b BC_0^b + \Omega_s BC_0^s$ is given by

$$\Omega_b c_0^b + \Omega_s c_0^s + q_0 \left[\Omega_b \left(d_0^b - (1-\delta) d_{-1}^b\right) + \Omega_s \left(d_0^s - (1-\delta) d_{-1}^s\right)\right] =$$

$$\Omega_b b_0^b + \Omega_s b_0^s + \omega_{c,0} \left(\Omega_b n_0^b + \Omega_s n_0^s\right) + \Pi_0^s$$

$$\iff Y_{c,0} + q_0 Y_{d,0} = \Omega_b b_0^b + \Omega_s b_0^s + \omega_{c,0} \left(\Omega_b n_0^b + \Omega_s n_0^s\right) + \left(Y_{c,0} - \omega_{c,0} N_{c,0}\right) + \left(q_0 Y_{d,0} - \omega_{c,0} N_{d,0}\right)$$

$$\iff 0 = \Omega_b b_0^b + \Omega_s b_0^s + \omega_{c,0} \left(\Omega_b n_0^b + \Omega_s n_0^s - N_{c,0} - N_{d,0}\right) \tag{A.1.50}$$

A similar argument for the final period implies,

$$0 = \omega_{c,1} \left(\Omega_b n_1^b + \Omega_s n_1^s - N_{c,1} - N_{d,1}\right) - R_0 \pi_{c,1}^{-1} \left(\Omega_b b_0^b + \Omega_s b_0^s\right) \tag{A.1.51}$$

From (A.1.50) and since $w_{c,0} > 0$, $\Omega_b n_0^b + \Omega_s n_0^s = N_{c,0} + N_{d,0} \iff 0 = \Omega_b b_0^b + \Omega_s b_0^s$.

From (A.1.51) and since $w_{c,1}, R_0 \pi_{c,1}^{-1} > 0$, $\Omega_b n_1^b + \Omega_s n_1^s = N_{c,1} + N_{d,1} \iff 0 = \Omega_b b_0^b + \Omega_s b_0^s$.

A.1.6 Proof of Proposition 1.6

Fix z_0, $(q_{-1}, \Delta_{c,-1}, \Delta_{d,-1})$, and $\left(R_{-1} b_{-1}^a, d_{-1}^a\right)_{a \in \{s,b\}}$.

First, it is a simple exercise to find an expression of $q_0 = f(\pi_{c,1})$ using the FOC for the intermediate firms' pricing in the final period $\Longrightarrow$ All the period 1 prices are functions of $(\pi_{d,0}, \pi_{c,0})$ via Implicit Function Theorem (IFT).

The borrower's optimal non-durable consumption level for period 0 is given implicitly via $g\left(\hat{c}_{b,0}^C\right)$, where $g\left(c_0^b\right)$ is defined in (A.1.22).

$\Longrightarrow \hat{c}_{b,0}^C = \hat{c}_{b,0}^C (\pi_{c,0}, \pi_{d,0}, \omega_{c,0})$ via IFT

$\Longrightarrow \hat{A}_b^C = \hat{A}_b^C (\pi_{c,0}, \pi_{d,0}, \omega_{c,0})$ via (A.1.12)-(A.1.18).

The saver's optimal durable consumption can be directly obtained

$\Longrightarrow \hat{c}_{s,0}^C = \hat{c}_{s,0}^C (\pi_{c,0}, \pi_{d,0}, \omega_{c,0})$

$\Longrightarrow \hat{A}_s^C = \hat{A}_s^C (\pi_{c,0}, \pi_{d,0}, \omega_{c,0})$ via the equivalent to (A.1.12)-(A.1.18).

In turn, this implies that the aggregate variables are given by $Y_{j,t} = Y_{j,t} (\pi_{c,0}, \pi_{d,0}, \omega_{c,0})$ for $a \in \{s, b\}$ and $j \in \{c, d\}$.

Thus using the FOC's for the optimal pricing for the intermediate firms in the first period and combining them with (1.3.5) we get:

$$\pi_{c,0} = (\phi_c)^{-1/(\epsilon_j - 1)} \left\{ 1 - \left[\Phi_c \left(1 + \beta_s \phi_c D_c \pi_{c,1}^{\epsilon_c - 1} \right) \left(\omega_{c,0} + \beta_s \phi_c D_c \pi_{c,1}^{\epsilon_c} \omega_{c,1} \right)^{-1} \right]^{(\epsilon_c - 1)} \right\}^{1/(\epsilon_c - 1)} \tag{A.1.52}$$

$$\pi_{d,0} = \left\{ \phi_d + \left[\Phi_d \, q_{-1} \, \pi_{c,0}^{-1} \left(1 + \beta_s \phi_d D_d \pi_{d,1}^{\epsilon_d - 1} \right) \left(\omega_{c,0} + \beta_s \phi_d D_d \pi_{d,1}^{\epsilon_d} \omega_{c,1} \right)^{-1} \right]^{(\epsilon_d - 1)} \right\}^{1/(\epsilon_d - 1)} \tag{A.1.53}$$

where, $\Phi_j \equiv (1 - \phi_j)^{1/(\epsilon_j - 1)} (\epsilon_j - 1) \, \epsilon_j^{-1}$, $D_j \equiv (Y_{j,1} \, c_{s,0}) (Y_{j,0} \, c_{s,1})^{-1}$.

$\implies \pi_{c,0} = \pi_{c,0} (\omega_{c,0})$ and $\pi_{d,0} = \pi_{d,0} (\omega_{c,0})$ by using the previous results and the IFT once again.

A.1.7 Proof of Proposition 1.7

Given the price vector $\hat{P}^{\text{NC}}$, for $t \in \{0, 1\}$ and $a \in \{b, s\}$, let $\left(\tilde{c}_t^a, \tilde{d}_t^a, \tilde{n}_t^a, \tilde{\zeta}_0^b \right)$ and $\left(\hat{c}_{a,t}^{\text{NC}}, \hat{d}_{a,t}^{\text{NC}}, \hat{n}_{a,t}^{\text{NC}} \right)$ denote the optimal allocations of an agent of type a in an economy with and without credit constraints, respectively. Consider, $N_0^{\text{E}} (\omega_{c,0}) = D_{N,0}^{\text{E}} (\omega_{c,0}) - S_{N,0}^{\text{E}} (\omega_{c,0})$, the excess demand function for labor in the initial period for economy $\text{E} \in \{\text{NC}, \text{C}\}$. Then $D_{N,0}^{\text{E}} (\omega_{c,0}) = N_{c,0}^{\text{E}} (\omega_{c,0}) + N_{d,0}^{\text{E}} (\omega_{c,0})$ refers to the aggregate demand of labor by the firms in each sector, and $S_{N,0}^{\text{E}} (\omega_{c,0}) = \Omega_s n_{s,0}^{\text{E}} (\omega_{c,0}) + \Omega_b n_{b,0}^{\text{E}} (\omega_{c,0})$ refers to the aggregate supply of labor by the agents. Given the price vector $\hat{P}^{\text{NC}}$, define $Q_0 \equiv \alpha^{-1} (1 - \alpha) \, \hat{R}_0^{\text{NC}} \left(\hat{R}_0^{\text{NC}} - (1 - \delta) \, \hat{\pi}_{d,1}^{\text{NC}} \right)^{-1}$. Thus one can write

$$D_{N,0}^{\text{NC}} \left(\hat{\omega}_{c,0}^{\text{NC}} \right) = \hat{\Delta}_{d,0}^{\text{NC}} \left[\Omega_s \left(\hat{d}_{s,0}^{\text{NC}} - (1 - \delta) \, d_{-1}^s \right) + \Omega_b \left(\hat{d}_{b,0}^{\text{NC}} - (1 - \delta) \, d_{-1}^b \right) \right] + \hat{\Delta}_{c,0}^{\text{NC}} \left(\Omega_s \hat{c}_{s,0}^{\text{NC}} + \Omega_b \hat{c}_{b,0}^{\text{NC}} \right)$$

$$\text{and } D_{N,0}^{\text{C}} \left(\hat{\omega}_{c,0}^{\text{NC}} \right) = \hat{\Delta}_{d,0}^{\text{NC}} \left[\Omega_s \left(\tilde{d}_0^s - (1 - \delta) \, d_{-1}^s \right) + \Omega_b \left(\tilde{d}_0^b - (1 - \delta) \, d_{-1}^b \right) \right] + \hat{\Delta}_{c,0}^{\text{NC}} \left(\Omega_s \tilde{c}_0^s + \Omega_b \tilde{c}_0^b \right).$$

The difference in the labor demand functions between the credit constraint and unconstrained economies is given by:

$$\begin{aligned} D_{N,0}^{\text{C}} - D_{N,0}^{\text{NC}} &= \hat{\Delta}_{d,0}^{\text{NC}} \left[\Omega_s \left(\tilde{d}_0^s - \hat{d}_{s,0}^{\text{NC}} \right) + \Omega_b \left(\tilde{d}_0^b - \hat{d}_{b,0}^{\text{NC}} \right) \right] + \hat{\Delta}_{c,0}^{\text{NC}} \left[\Omega_s \left(\tilde{c}_0^s - \hat{c}_{s,0}^{\text{NC}} \right) + \Omega_b \left(\tilde{c}_0^b - \hat{c}_{b,0}^{\text{NC}} \right) \right] \\ &= \Omega_b \left[\hat{\Delta}_{d,0}^{\text{NC}} \left(\tilde{d}_0^b - \hat{d}_{b,0}^{\text{NC}} \right) + \left(\tilde{c}_0^b - \hat{c}_{b,0}^{\text{NC}} \right) \right] + \Omega_s \left[\hat{\Delta}_{c,0}^{\text{NC}} + \left(\hat{q}_0^{\text{NC}} \right)^{-1} \hat{Q}_0 \hat{\Delta}_{c,0}^{\text{NC}} \right] \left(\tilde{c}_0^s - \hat{c}_{s,0}^{\text{NC}} \right); \end{aligned} \tag{A.1.54}$$

where I have used equation (A.1.1) to substitute for d_0^s.

Similarly using equation (A.1.15), one can write the difference in the labor supply functions between the credit constraint and unconstrained economies as:

$$S_{N,0}^{\mathrm{C}} - S_{N,0}^{\mathrm{NC}} = \alpha \hat{\omega}_{c,0}^{\mathrm{NC}} \left[\nu_s^{-1} \Omega_s \left(\hat{c}_{s,0}^{\mathrm{NC}} - \tilde{c}_0^s \right) \left(\hat{c}_{s,0}^{\mathrm{NC}} \, \tilde{c}_0^s \right)^{-1} + \nu_b^{-1} \Omega_b \left(\hat{c}_{b,0}^{\mathrm{NC}} - \tilde{c}_0^b \right) \left(\hat{c}_{b,0}^{\mathrm{NC}} \, \tilde{c}_0^b \right)^{-1} \right]. \tag{A.1.55}$$

Given Q_0^a, Q_1^a, $\overline{W}_{-1}^s$, K_R^a, K_C^a, $A_{c,t}$, $A_{d,t}$, B_ζ, D_ζ, Δ_{K,d_0}, Δ_{K,c_1}, and Δ_K; proceed as in Proposition 1.1 and let $\bar{A}_{d_0}^b \equiv \left[A_{d,0} - \left(\hat{R}_0^{\mathrm{NC}} \right)^{-1} \hat{\pi}_{c,1}^{\mathrm{NC}} \left(1 - \delta \right) A_{d,1} \right]$,
$\bar{A}_{c_1}^b \equiv \left(\hat{R}_0^{\mathrm{NC}} \right)^{-1} \hat{\pi}_{c,1}^{\mathrm{NC}} \left[A_{c,1} + \left(\alpha \, \hat{q}_1^{\mathrm{NC}} \right)^{-1} \left(1 - \alpha \right) \left(1 + \kappa_b \right) A_{d,1} \right]$,
and $W_F^s \equiv \Omega_b \Omega_s^{-1} \left(c_0^b A_{c,0} + d_0^b \overline{A}_{d_0}^b + c_1^b \overline{A}_{c_1}^b \right)$.

Hence, for the allocations $\left(\tilde{c}_0^b, \tilde{d}_0^b, \tilde{c}_1^b \right)$, $\left(\hat{c}_{a,0}^{\mathrm{NC}}, \hat{d}_{a,}^{\mathrm{NC}}, \hat{c}_{a,1}^{\mathrm{NC}} \right)$ and using equations (A.1.1), (A.1.2), (A.1.13), (A.1.14), one can write $\widetilde{W}_F^s = \Omega_b \Omega_s^{-1} \tilde{c}_0^b \left(\hat{K}_R^b - \hat{K}_C^b + \Delta_K \right)$ and $\overline{W}_F^s = \Omega_b \Omega_s^{-1} \hat{c}_{b,0}^{\mathrm{NC}} \left(\hat{K}_R^b - \hat{K}_C^b \right)$.

$$\Longrightarrow \overline{W}_F^s - \widetilde{W}_F^s = \Omega_b \Omega_s^{-1} \left[\left(\hat{K}_R^b - \hat{K}_C^b \right) \left(\hat{c}_{b,0}^{\mathrm{NC}} - \tilde{c}_0^b \right) - \tilde{c}_0^b \Delta_K \right] \tag{A.1.56}$$

Now, given $\hat{P}^{\mathrm{NC}}$, $\overline{W}_{-1}^s$, and since $\left(K_R^b - K_C^b \right) \geq 0$, we have

1. Case 1: $\hat{c}_{s,0}^{\mathrm{NC}} \geq \tilde{c}_0^s \Longleftrightarrow \overline{W}_F^s \geq \widetilde{W}_F^s \Longleftrightarrow \hat{c}_{b,0}^{\mathrm{NC}} \geq \tilde{c}_0^b \left(K_R^b - K_C^b \right)^{-1} \left(K_R^b - K_C^b + \Delta_K \right)$. The first equivalence relation follows from (A.1.11) and the second from (A.1.56). Then from Proposition 1.4, equations (A.1.54), (A.1.55), and noting that $N_0^{\mathrm{NC}} \left(\hat{\omega}_{c,0}^{NC} \right) = 0$; it follows that $N_0^{\mathrm{C}} \left(\hat{\omega}_{c,0}^{NC} \right) \leq 0$.

2. Case 2: $\hat{c}_{s,0}^{\mathrm{NC}} < \tilde{c}_0^s \Longleftrightarrow \overline{W}_F^s < \widetilde{W}_F^s \Longleftrightarrow \hat{c}_{b,0}^{\mathrm{NC}} < \tilde{c}_0^b \left(K_R^b - K_C^b \right)^{-1} \left(K_R^b - K_C^b + \Delta_K \right)$. Note that

   (a) From (A.1.11) it follows that $0 \leq \left(\tilde{c}_0^s - \hat{c}_{s,0}^{\mathrm{NC}} \right) \leq \left(\widetilde{W}_F^s - \overline{W}_F^s \right) \left(K_C^s \right)^{-1}$.

   (b) From (A.1.56), $\left(\widetilde{W}_F^s - \overline{W}_F^s \right) > 0$ is not a function of κ_s and $\lim_{\kappa_s \to +\infty} K_C^s = +\infty$. Furthermore, $\left(\tilde{d}_0^b - \hat{d}_{b,0}^{\mathrm{NC}} \right)$ and $\left(\tilde{c}_0^b - \hat{c}_{b,0}^{\mathrm{NC}} \right)$ are not functions of κ_s either.

   (c) $\overline{W}_{-1}^s \geq \Omega_s^{-1} \hat{c}_{b,0}^{\mathrm{NC}} \left[\Omega_s K_C^s + \Omega_b \left(K_C^b - K_R^b \right) \right] \Longleftrightarrow \left(\overline{W}_{-1}^s + \overline{W}_F^s \right) \left(K_C^s \right)^{-1} \geq \hat{c}_{b,0}^{\mathrm{NC}}$. So that given (A.1.11) $\Longrightarrow \hat{c}_{s,0}^{\mathrm{NC}} \geq \left(\overline{W}_{-1}^s + \overline{W}_F^s \right) \left(K_C^s \right)^{-1} \geq \hat{c}_{b,0}^{\mathrm{NC}}$.

   (d) From Proposition 1.4 and given $\Delta_K > 0$
   $\Longrightarrow \overline{W}_{-1}^s \geq \Omega_s^{-1} \hat{c}_{b,0}^{\mathrm{NC}} \left[\Omega_s K_C^s + \Omega_b \left(K_C^b - K_R^b \right) \right] \geq \Omega_s^{-1} \tilde{c}_0^b \left[\Omega_s K_C^s + \Omega_b \left(K_C^b - K_R^b - \Delta_K \right) \right]$
   $\Longrightarrow \tilde{c}_0^s \geq \tilde{c}_0^b$.

   Hence, from observations (a), (b), Proposition 1.4 and equation (A.1.54), it follows that for κ_s large enough

$D^{\text{C}}_{N,0}\left(\hat{\omega}^{\text{NC}}_{c,0}\right) - D^{\text{NC}}_{N,0}\left(\hat{\omega}^{\text{NC}}_{c,0}\right) \leq 0.$

Similarly, from observations (a)-(d), Proposition 1.4 and equation (A.1.55), for κ_s large enough

$$S^{\text{C}}_{N,0}\left(\hat{\omega}^{\text{NC}}_{c,0}\right) - S^{\text{NC}}_{N,0}\left(\hat{\omega}^{\text{NC}}_{c,0}\right) \geq \left(\frac{\alpha\hat{\omega}^{\text{NC}}_{c,0}}{\hat{c}^{\text{NC}}_{b,0}\tilde{c}^{b}_{0}}\right)\left[\nu_s^{-1}\Omega_s\left(\overline{W}^{s}_{F} - \widetilde{W}^{s}_{F}\right)(K^{s}_{C})^{-1} + \nu_b^{-1}\Omega_b\left(\hat{c}^{\text{NC}}_{b,0} - \tilde{c}^{b}_{0}\right)\right] \geq 0.$$

So that $N^{\text{C}}_{0}\left(\hat{\omega}^{NC}_{c,0}\right) - N^{\text{NC}}_{0}\left(\hat{\omega}^{NC}_{c,0}\right) \leq 0$; and noting that $N^{\text{NC}}_{0}\left(\hat{\omega}^{NC}_{c,0}\right) = 0$, the desired result follows.

A.3 Appendix To Chapter 3

A.3.1 Additional Tables

Table 5.1: Correlation Coefficients for Credit Variables

	$C\&I_S$	$Mort_S$	NCC_S	CC_S	$C\&I_D$	$Mort_D$	HH_D
$C\&I_S$	1.000	0.642	0.661	0.497	-0.643	-0.042	-0.338
$Mort_S$	0.642	1.000	0.761	0.558	-0.376	-0.197	-0.457
NCC_S	0.661	0.761	1.000	0.879	-0.480	-0.062	-0.465
CC_S	0.497	0.558	0.879	1.000	-0.368	0.078	-0.342
$C\&I_D$	-0.643	-0.376	-0.480	-0.368	1.000	-0.175	0.354
$Mort_D$	-0.042	-0.197	-0.062	0.078	-0.175	1.000	0.449
HH_D	-0.338	-0.457	-0.465	-0.342	0.354	0.449	1.000

Note: Based on quarterly data for the period 1990:1-2012:2

Table 5.2: SLOOS Principal Component Loading Factors

	Credit Supply Block			
	PC_S^1	PC_S^2	PC_S^3	PC_S^4
$C\&I_S$	0.608	0.734	-0.280	-0.118
$Mort_S$	0.454	-0.100	0.842	-0.275
NCC_S	0.481	-0.271	-0.018	0.834
CC_S	0.440	-0.615	-0.462	-0.463
	Credit Demand Block			
	PC_D^1	PC_D^2	PC_D^3	
$C\&I_D$	-0.161	0.945	-0.285	
$Mort_D$	0.918	0.037	-0.395	
HH_D	0.362	0.325	0.873	

Note: Sample period 1995:10-2012:6.

Table 5.3: Effects of High-Frequency Instrument on the First Stage Residuals of the Monetary Policy Indicator for Alternative Specifications of the VAR_1

	VAR_1^*			
	Spreads	$\widehat{\text{Spreads}}$	SLOOS	$\widehat{\text{SLOOS}}$
Constant	0.010	0.010	0.011	0.011
	(0.006-0.014)	(0.006-0.013)	(0.006-0.014)	(0.005-0.015)
FF_3	0.800	0.785	0.817	0.831
	(0.450-1.082)	(0.473-0.996)	(0.468-1.157)	(0.439-1.184)
Observations	258	258	258	258
Adj. R^2	0.065	0.065	0.058	0.058
F-statistic	16.713	21.744	15.081	12.736

	$VAR_1^\dagger$			
	Spreads	$\widehat{\text{Spreads}}$	SLOOS	$\widehat{\text{SLOOS}}$
Constant	0.010	0.010	0.011	0.011
	(0.006-0.014)	(0.006-0.013)	(0.006-0.014)	(0.006-0.015)
FF_3	0.800	0.785	0.808	0.834
	(0.445-1.056)	(0.471-0.969)	(0.469-1.084)	(0.452-1.161)
Observations	258	258	258	258
Adj. R^2	0.065	0.065	0.058	0.063
F-statistic	16.713	21.744	16.826	14.654

	$VAR_1^\ddagger$			
	Spreads	$\widehat{\text{Spreads}}$	SLOOS	$\widehat{\text{SLOOS}}$
Constant	0.010	0.010	0.010	0.011
	(0.006-0.014)	(0.007-0.014)	(0.006-0.013)	(0.006-0.014)
FF_3	0.800	0.785	0.784	0.814
	(0.444-1.077)	(0.480-0.999)	(0.463-1.021)	(0.445-1.126)
Observations	258	258	258	258
Adj. R^2	0.065	0.065	0.059	0.063
F-statistic	16.713	21.744	17.241	15.695

Note: Sample period 1990:1-2012:6. 90 percent confidence intervals in parenthesis.
1YR is used as the monetary policy indicator.
The surprise in the three month ahead fed funds future (FF_3) is used as instrument.
* SLOOS includes only the "main" principal component of credit supply block.
† SLOOS includes only the "main" principal component of credit demand block.
‡ SLOOS includes the credit demand and supply "main" principal components.

A.4 Appendix To Chapter 4

A.4.1 Proof of Proposition 4.1

Given the recursive formulation of the household's problem given by equation 4.3.7, the FOC's for c_t^{NR}, l_t^{NR}, c_t^{R}, and l_t^{R} are given by

$$
\begin{aligned}
\underline{c_t^{\text{NR}}} &: \ u_c\left(c_t^{\text{NR}}, l_t^{\text{NR}}\right)(1-\sigma) - \kappa^{\text{NR}} = 0 \\
\underline{l_t^{\text{NR}}} &: \ u_l\left(c_t^{\text{NR}}, l_t^{\text{NR}}\right)(1-\sigma) + \kappa^{\text{NR}}\left[w_t z - \tau_t\prime\left(w_t z l_t^{\text{NR}}\right)\right] = 0 \\
\underline{c_t^{\text{R}}} &: \ u_c\left(c_t^{\text{R}}, l_t^{\text{R}}\right)\sigma - \kappa^{\text{R}} = 0 \\
\underline{l_t^{\text{R}}} &: \ u_l\left(c_t^{\text{R}}, l_t^{\text{R}}\right)\sigma + \kappa^{\text{R}}\left[w_t z - \tau_t\prime\left(w_t z l_t^{\text{R}}\right)\right] = 0;
\end{aligned}
$$

where κ^{NR} is the multiplier of the budget constraint conditional on no renegotiation, κ^{R} is the multiplier of the budget constraint conditional on renegotiation, and $\tau_t\prime(\cdot)$ is the derivative of the tax function. Given the assumptions that $\tau_t(x) = \tau_t^l + T_t$, $u(c,l) = \frac{[c-g(l)]^{1-\gamma}-1}{1-\gamma}$, with $\gamma > 1$, and $g(l) = \frac{\psi z l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$; the previous conditions imply $l_t^{\text{NR}} = l_t^{\text{R}} = \left(\frac{(1-\tau_t^l)w_t}{\psi}\right)^{\nu}$.

Using the budget constraint, it is easy to see that $c_t^{\text{NR}} = \left(1-\tau_t^l\right) w_t z l_t^{\text{NR}} + r_t b + T_t$. Similarly, for the renegotiation case $c_t^{\text{R}} = \left(1-\tau_t^l\right) w_t z l_t^{\text{R}} + r_t b + T_t - (b'-b) - \phi$. Thus the desired results follow.

A.4.2 Proof of Proposition 4.2

I first show that there exist a $\underline{\text{b}}$ such that $b' > \underline{\text{b}}$ for any feasible b'. Given the bargaining problem (4.3.6), if the constraint $b' > \underline{\text{b}}$ is binding then the lower bound is trivial. Assume that $\underline{\text{b}}$ is sufficiently small so that this constraint never binds. Given $u(c,l) = \frac{[c-g(l)]^{1-\gamma}-1}{1-\gamma}$, with $\gamma > 1$, then $\lim_{c\to\infty} u(c,l) = 0$. Therefore, the household's surplus is bounded above

$$CS_t = \left[V_t(b',z;\Psi_t) + u(c_t^{\text{R}}, l_t^*)\right] - \left[V_t(b,z;\Psi_t) + u(c_t^{\text{NR}}, l_t^*)\right] \leq V_t(b',z;\Psi_t) - \left[V_t(b,z;\Psi_t) + u(c_t^{\text{NR}}, l_t^*)\right].$$

Given that the value function is continuous and increasing in the asset holdings, as long as $\lim_{b'\to-\infty} V_t(b',z;\Psi_t) < V_t(b,z;\Psi_t) + u(c_t^{\text{NR}}, l_t^*)$, there exists $\underline{\text{b}}$ such that $V_t(\underline{\text{b}},z;\Psi_t) - \left[V_t(b,z;\Psi_t) + u(c_t^{\text{NR}}, l_t^*)\right] < 0$. Thus any feasible b' must satisfy $b' > \underline{\text{b}}$.

I next show that there exists a $\bar{b}$ such that $b' < \bar{b}$ for any feasible b'. Given that $BS_t = \phi + r_t(b'-b) \geq 0$, let $\phi = -r_t(b'-b)$ so that $c_t^{\text{R}}(b') = \left(1-\tau_t^l\right) w_t z l_t^{\text{R}} + r_t b + T_t - (b'-b)(1-r_t)$.

Note that due to the GHH utility specification, $c_t^{\mathrm{R}}(b')$ is bounded below; $c_t^{\mathrm{R}}(b') \geq g(l_t^*)$. Define $\bar{b}$ so that $c_t^{\mathrm{R}}(\bar{b}) = g(l_t^*)$. Assume further that $r_t < 1$ so that $c_t^{\mathrm{R}}(b')$ is decreasing in b'. Thus, $u\left(c_t^{\mathrm{R}}(b'), l_t^*\right) = -\infty$ for any $b' \geq \bar{b}$. As long as $V_t(\bar{b}, z; \Psi_t) < \infty$, the desired result follows by noting that $CS_t \geq 0$ for any feasible b'.

That $b \in \left(\underline{\mathrm{b}}, \bar{b}\right)$ follows by noting that b is always a feasible choice since $CS_t = BS_t = 0$ whenever $b' = b$.

Finally, I derive expression (4.3.8). The FOC of the contracting problem (4.3.6) with respect to ϕ implies that

$$\theta \left(\frac{CS_t}{BS_t}\right)^{1-\theta} - (1-\theta) u_c\left(c_t^{\mathrm{R}}, l_t^*\right) \left(\frac{CS_t}{BS_t}\right)^{-\theta} = 0. \tag{A.4.1}$$

Given the definitions of CS_t and BS_t, equation (A.4.1) can be written as

$$\begin{aligned}
&[\phi + r_t(b'-b)]\, u_c\left(c_t^{\mathrm{R}}, l_t^*\right) - \tfrac{\theta}{1-\theta} u\left(c_t^{\mathrm{R}}, l_t^*\right) = \tfrac{\theta}{1-\theta}\left[V_t(b', z; \Psi_t) - V_t(b, z; \Psi_t) - u\left(c_t^{\mathrm{NR}}, l_t^*\right)\right] \\
&\Longrightarrow \left[c_t^{\mathrm{R}} - g(l^*)\right]^{-\gamma} \left[\phi + r_t(b'-b)\right] + \left[c_t^{\mathrm{R}} - g(l^*)\right]^{1-\gamma} = \widetilde{CS}_t \\
&\Longrightarrow \left[c_t^{\mathrm{R}} - g(l^*)\right]^{-\gamma} \left[c_t^{\mathrm{NR}} - c_t^{\mathrm{R}} - (1-r_t)(b'-b)\right] + \left[c_t^{\mathrm{R}} - g(l^*)\right]^{1-\gamma} = \widetilde{CS}_t \\
&\Longrightarrow \left[c_t^{\mathrm{R}} - g(l^*)\right]^{-\gamma} \widetilde{BS}_t = \widetilde{CS}_t \\
&\Longrightarrow \phi = \left[c_t^{\mathrm{NR}} - (b'-b) - g(l^*)\right] - \left(\frac{\widetilde{BS}_t}{\widetilde{CS}_t}\right)^{1/\gamma};
\end{aligned}$$

where $\widetilde{CS}_t \equiv V_t(b', z; \Psi_t) - V_t(b, z; \Psi_t) - u(c_t^{\mathrm{NR}}, l^*) - (1-\gamma)^{-1}$ and $\widetilde{BS}_t \equiv \left[c_t^{\mathrm{NR}} - (1-r_t)(b'-b) - g(l_t^*)\right]$. The second line uses the assumptions that $u(c,l) = \frac{[c-g(l)]^{1-\gamma}-1}{1-\gamma}$ and $-\left(\frac{\theta}{1-\theta}\right)\left(\frac{1}{1-\gamma}\right) = 1$. Additionally, note that I use the fact that $c_t^{\mathrm{R}} = c_t^{\mathrm{NR}} - (b'-b) - \phi$.

A.4.3 Proof of Proposition 4.4

I first derive expression (4.3.13). The FOC of the contracting problem (4.3.6) with respect to b' implies that

$$\theta\, r_t \left(\frac{CS_t}{BS_t}\right)^{1-\theta} + (1-\theta)\left[\partial_b V_t(b', z; \Psi_t) - u_c(c_t^{\mathrm{R}}, l^*)\right] \left(\frac{CS_t}{BS_t}\right)^{-\theta} = 0. \tag{A.4.2}$$

Using equation (A.4.1) to substitute for $\frac{CS_t}{BS_t}$ one gets the desired result; $\frac{\partial_b V_t(b', z; \Psi_t)}{u_c(c_t^{\mathrm{R}}, l_t^*)} = 1 - r_t$.

Given that $u(\cdot)$ is concave and differentiable, the assumption that $V_t(b, z; \Psi_t)$ is concave and differentiable ensures that the FOC's given by equations (A.4.1) and (A.4.2) are necessary and sufficient to characterize the solution to the contracting problem.

A.4.4 Proof of Proposition 4.5

I show that $\frac{\partial b'(b,z)}{\partial b} \geq 0$. Note that the contracting problem can be solved in two steps. First solve for $b'(b,z)$ by combining equations (4.3.8) and (4.3.13). Second, use (4.3.8) to get $\phi(b'(b,z),b,z)$. That is, the optimal fee in the contract can be seen as a function of b', b, and z.

Differentiating (4.3.13) with respect to b one gets

$$\frac{\partial b'}{\partial b} = \left[1 + r_t - \frac{\partial \phi}{\partial b}\right] \cdot \left[\frac{V_{bb}(b', z; \Psi)}{u_{cc}(c_t^{\mathrm{R}}, l_t^*)} + 1 + \frac{\partial \phi}{\partial b'}\right]^{-1}; \tag{A.4.3}$$

where $\frac{\partial \phi}{\partial b'}$ and $\frac{\partial \phi}{\partial b}$ can be obtained from equation (A.4.1) as

$$\frac{\partial \phi}{\partial b'} = -\left[r_t - \theta \cdot CS_t \cdot \left(\frac{u_{cc}(c_t^{\mathrm{R}}, l_t^*)}{u_c(c_t^{\mathrm{R}}, l_t^*)}\right)\right] \cdot \left[1 - \theta \cdot CS_t \cdot \left(\frac{u_{cc}(c_t^{\mathrm{R}}, l_t^*)}{u_c(c_t^{\mathrm{R}}, l_t^*)}\right)\right]^{-1}$$
$$\frac{\partial \phi}{\partial b} = 1 + r_t - \gamma^{-1} \cdot \left(\frac{\widetilde{BS}_t}{\widetilde{CS}_t}\right)^{1/\gamma} \cdot \left(\widetilde{CS}_t\right)^{-2} \cdot \left(\frac{\theta}{1-\theta}\right) \cdot \left[V_b(b, z; \Psi_t) + u_c(c_t^{\mathrm{NR}}, l_t^*) r_t\right]$$

Given that $u_{cc}(\cdot) < 0$, $u_c(\cdot) > 0$, $CS_t \geq 0$, $\gamma > 1$, and $\left(\frac{\widetilde{BS}_t}{\widetilde{CS}_t}\right) \geq 0$:

1. As long as $r_t < 1$; it follows $\frac{\partial \phi}{\partial b'} \in (-1, 0)$.

2. As long as the value function $V(\cdot)$ is an increasing function of b; it follows that $\left(1 + r_t - \frac{\partial \phi}{\partial b}\right) \geq 0$.

Therefore, from equation (A.4.3) it follows that $\frac{\partial b'(b,z)}{\partial b} \geq 0$ as long as the flow utility and value functions are concave (i.e. $V_{bb}(\cdot), u_{cc}(\cdot) < 0$).

A.4.5 Proof of Proposition 4.6

Trivial. Use (4.3.8) to substitute for ϕ in equation (4.3.13). The resulting expression gives b' as an implicit function of r_t. Appealing to the Implicit Function Theorem, differentiate this expression with respect to r_t. A little bit of algebra and rearrangement of the terms yields expression (4.4.1)

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Committee: Bulent Guler (Co-chair), Juan Carlos Hatchondo (Co-chair), Amanda M. Michaud, Eric M. Leeper
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References:

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Fields of Specialization:

Research Fields: Macroeconomics, Monetary Economics, Financial Economics, Macroeconometrics, Computational Macroeconomics

Teaching Fields: Introductory/Intermediate Microeconomics, Macroeconomics, and Econometrics for undergraduate level. Any course related to my research fields for undergraduate and graduate level.

Teaching Experience:

Spring, 2016 Spring, 2015 Fall, 2014 Summer, 2013	Introduction to Microeconomics, Indiana University, Associate Instructor
Summer, 2016 Fall, 2015 Summer, 2015 Summer, 2014	Intermediate Macroeconomic Theory, Indiana University, Associate Instructor
Summer, 2014	Matlab Mini-Course, Indiana University, Instructor
Fall, 2016	Introduction to Microeconomics, Indiana University, Graduate Teaching Assistant for Professor Gerhard Glomm
Fall, 2012 Summer, 2012	Introduction to Microeconomics, Indiana University, Graduate Teaching Assistant for Professor James K. Self

Research Experience:

Spring, 2014 Fall, 2013 Spring, 2013	Indiana University, Research Assistant for Professor Eric M. Leeper
Fall, 2010 Spring, 2011	University of Pennsylvania, Research Assistant for Professor Susan M. Wachter

Conference Presentations:

2017	13th Annual Workshop on Macroeconomics Research at Liberal Arts Colleges (discussant)
2016	53rd Annual Missouri Valley Economic Association Conference (presenter, discussant, and session chair), Midwest Macro Meetings, Purdue University
2015	Workshop on Monetary Policy and the Distribution of Income and Wealth, Federal Reserve Bank of St. Louis

2014	10th Annual Jordan River Conference, Indiana University (presenter)

Honors, Scholarships, and Fellowships:

2015	Daniel J. Duesterberg Book Award for Policy Research, Indiana University
2015	W. Phillip Saunders Award for Teaching, Indiana University
2011-2012	Department of Economics Graduate Fellowship, Indiana University
2009-2010	Lawrence R. Klein Fellowship, University of Pennsylvania

Working Papers:

"Monetary Policy Revisited: Heterogeneous Bank Pass-Through of Credit Expansions" (Job Market Paper)

"An Empirical Assessment of the Transmission of Monetary Policy"

"Monetary Policy and Credit Constraints"

"Credit Constraints in a New Keynesian Framework: A Simple Theoretical Analysis"

Computer Skills:

Fortran, Matlab, Stata, LaTeX

Languages:

Spanish (native), English (fluent)